FAILURE MODES FOR PROOFS

Moon Duchin
Tufts University

QUESTIONS, METHODS, GOALS

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A group theory of group theory: Collaborative mathematics and the 'uninvention' of a 1000-page proof

Alma Steingart

Abstract

Over a period of more than 30 years, more than 100 mathematicians worked on a project to classify mathematical objects known as finite simple groups. The Classification, when officially declared completed in 1981, ranged between 300 and 500 articles and ran somewhere between 5,000 and 10,000 journal pages. Mathematicians have hailed the project as one of the greatest mathematical achievements of the 20th century, and it surpasses, both in scale and scope, any other mathematical proof of the 20th century. The history of the Classification points to the importance of face-to-face interaction and close teaching relationships in the production and transformation of theoretical knowledge. The techniques and methods that governed much of the work in finite simple group theory circulated via personal, often informal, communication, rather than in published proofs. Consequently, the printed proofs that would constitute the Classification Theorem functioned as a sort of shorthand for and formalization of proofs that had already been established during personal interactions among mathematicians. The proof of the Classification was at once both a material artifact and a crystallization of one community's shared practices, values, histories, and expertise. However, beginning in the 1980s, the original proof of the Classification faced the threat of 'uninvention'. The papers that constituted it could still be found scattered throughout the mathematical literature, but no one other than the dwindling community of group theorists would know how to find them or how to piece them together. Faced with this problem, finite group theorists resolved to produce a 'second-generation proof' to streamline and centralize the Classification. This project highlights that the proof and the community of finite simple groups theorists who produced it were co-constitutive–one formed and reformed by the other.

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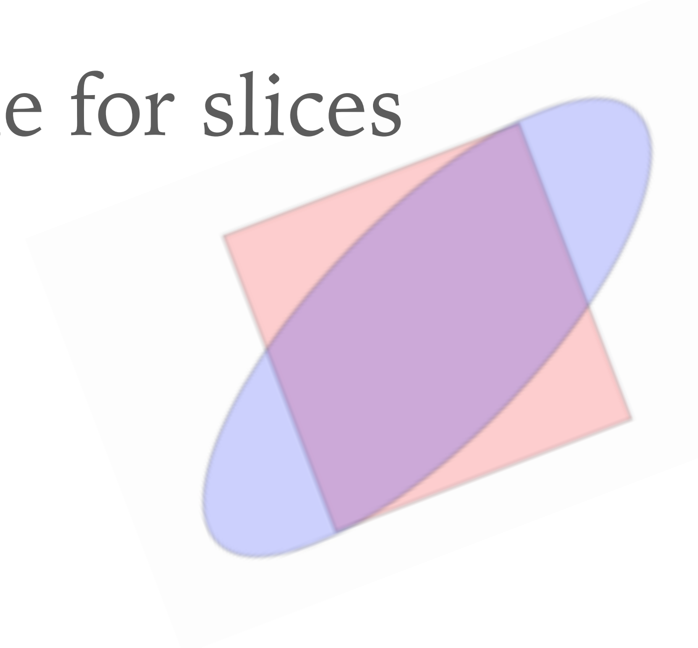

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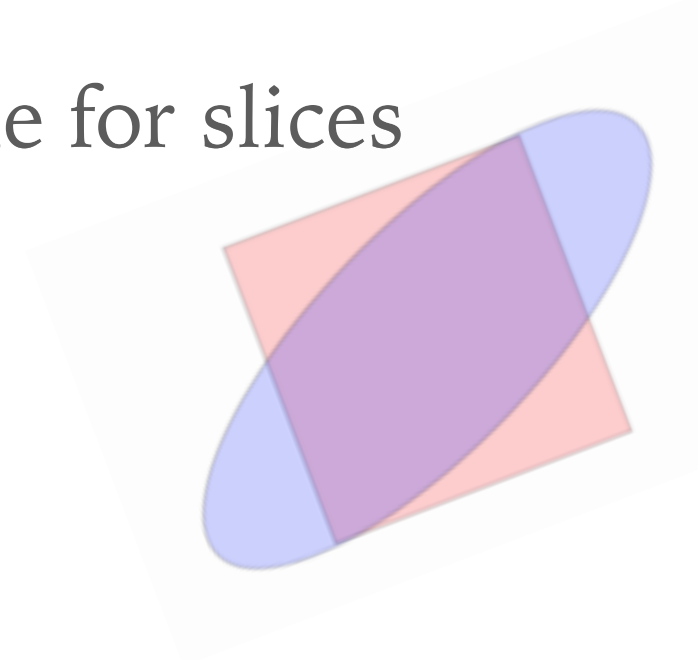

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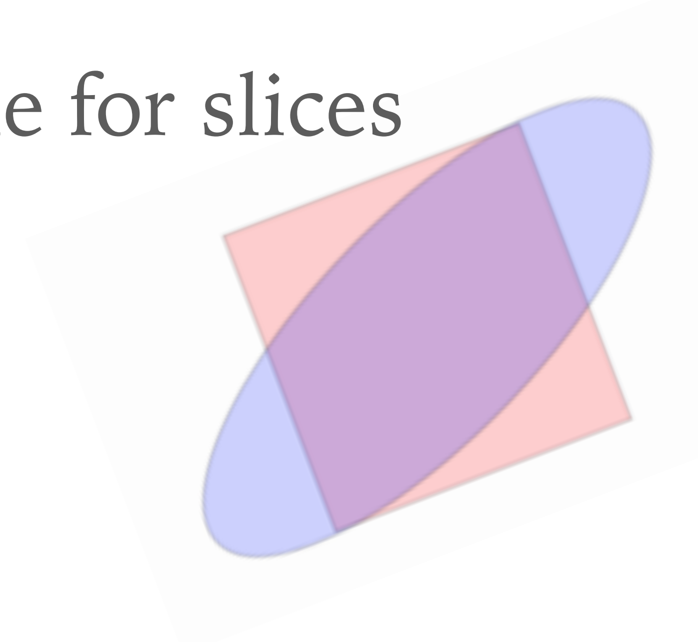

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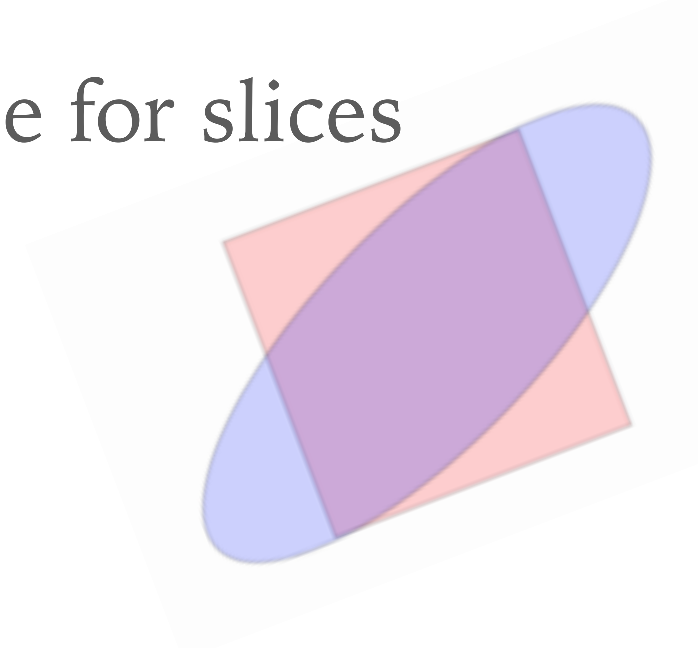

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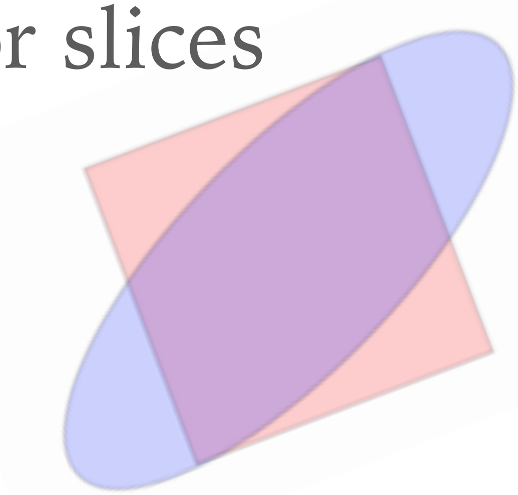

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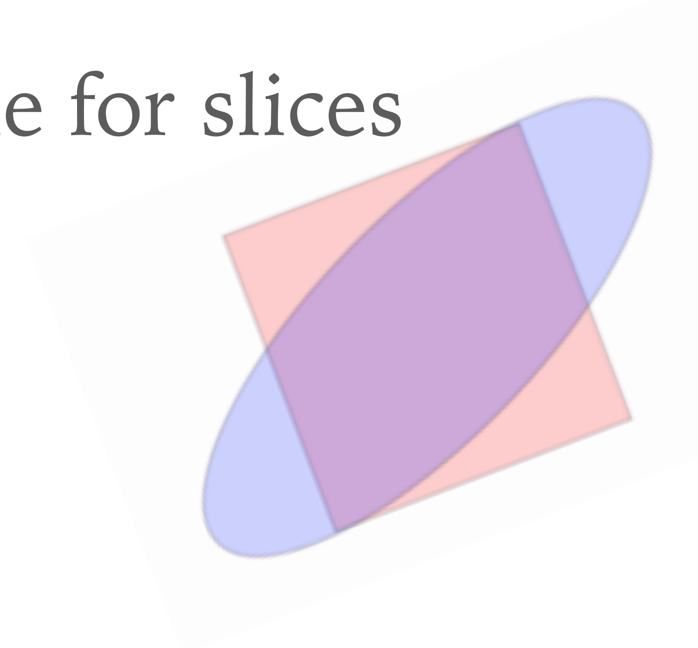

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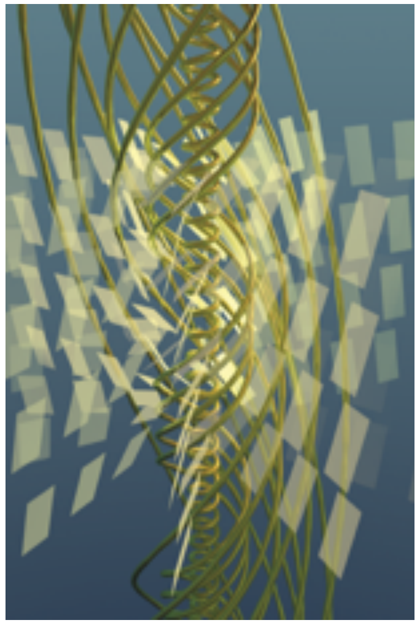

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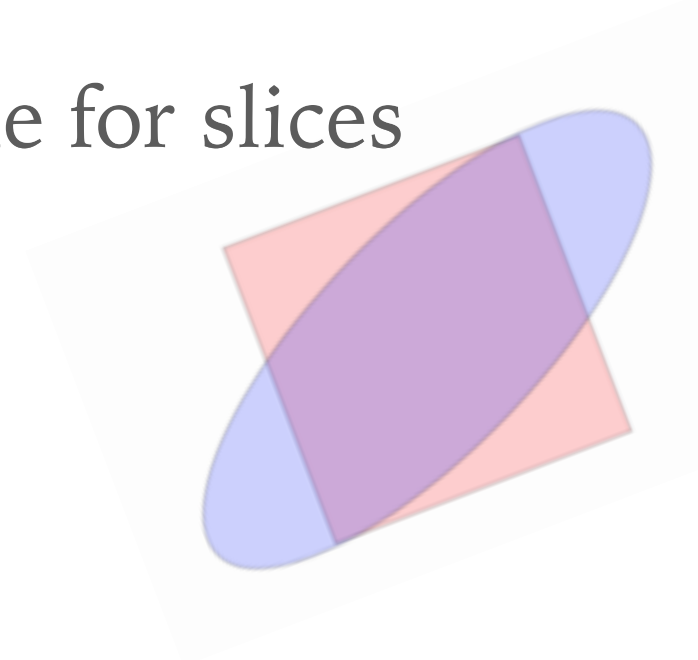

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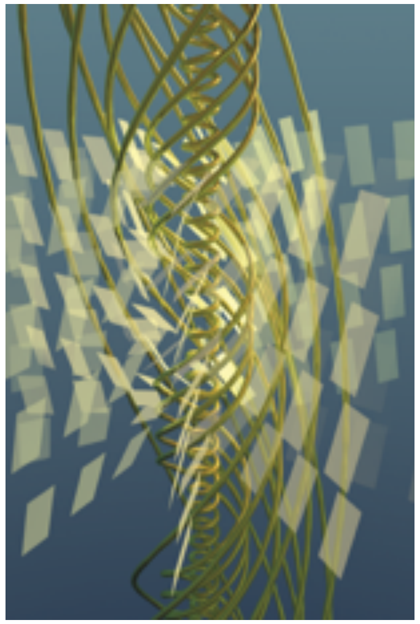

  - Osin to Sohrabi: your theorem can't be right because it conflicts with my examples

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  - M.Aschenbrenner: "What Tom [Scanlon] did [on Pop's conjecture] is not a complete waste; many of the ideas can be rescued. And there's still the possibility that his approach can be made to work—it's just, no one knows how…"

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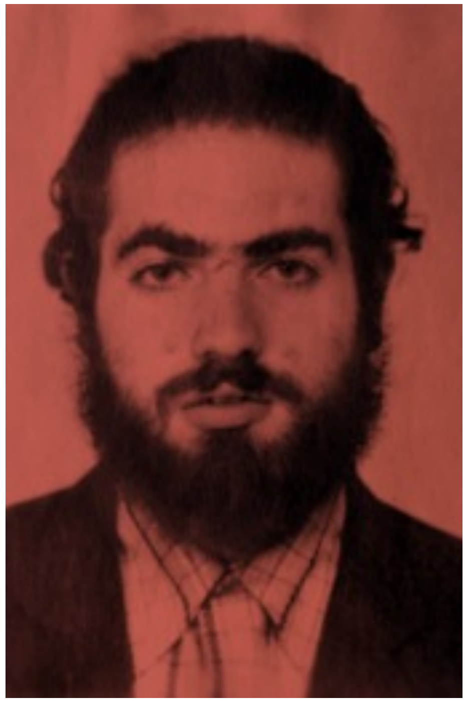

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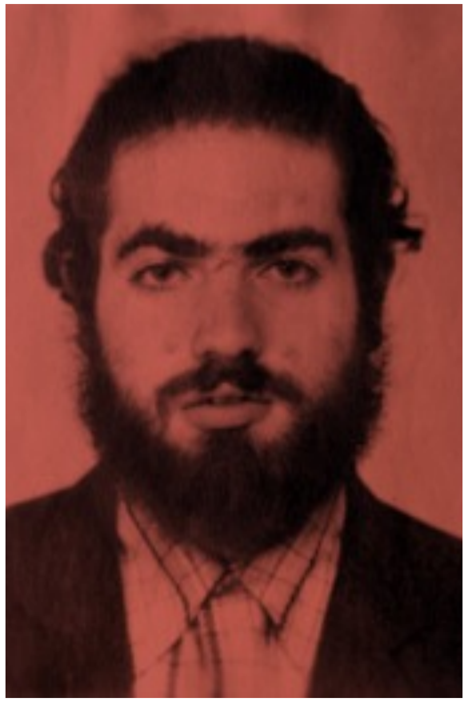

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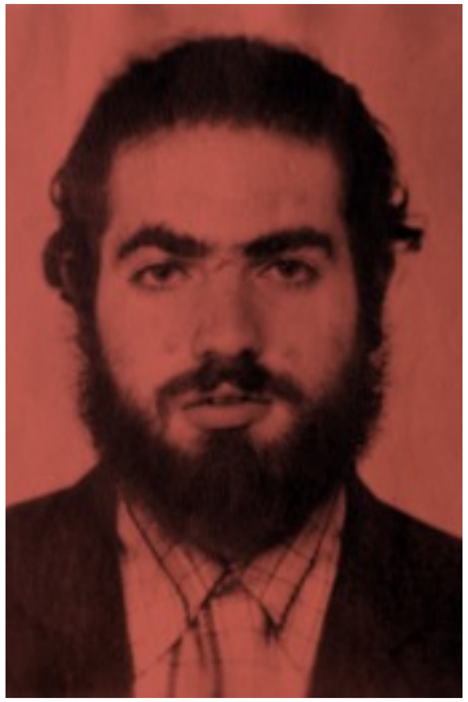
  Kleiner-Lott: “Regarding the proofs, the papers [28, 29] contain some incorrect statements and incomplete arguments… We did not find any serious problems, meaning problems that cannot be corrected using the methods introduced by Perelman.”



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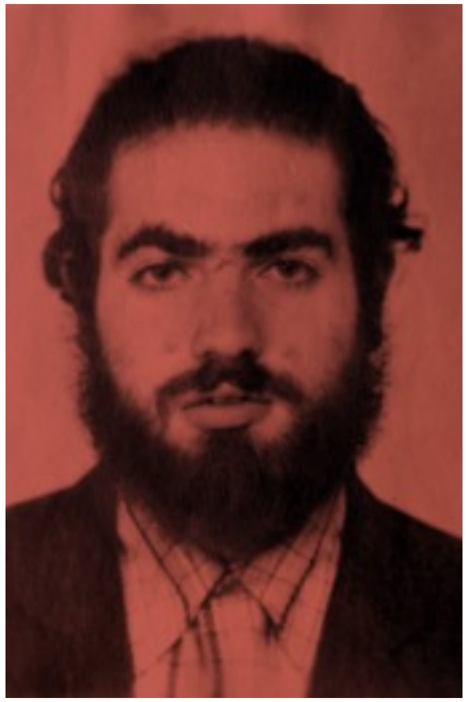

- Perfectability in principle (see Lakatos)

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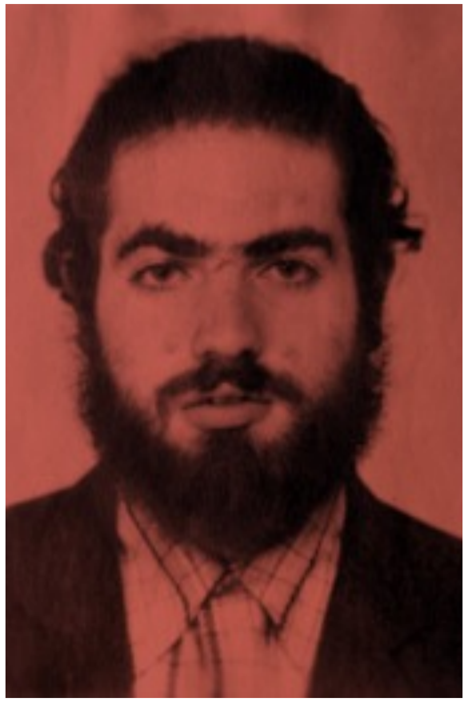

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  Most experts are convinced that the proof is essentially correct; any errors which occur are expected to be minor oversights or local errors which can be corrected by the methods that have been developed in the process of completing the classification. More importantly, no error is expected to change the end result, that is, to lead to new simple groups. (Feit, 1983: 120)[40]

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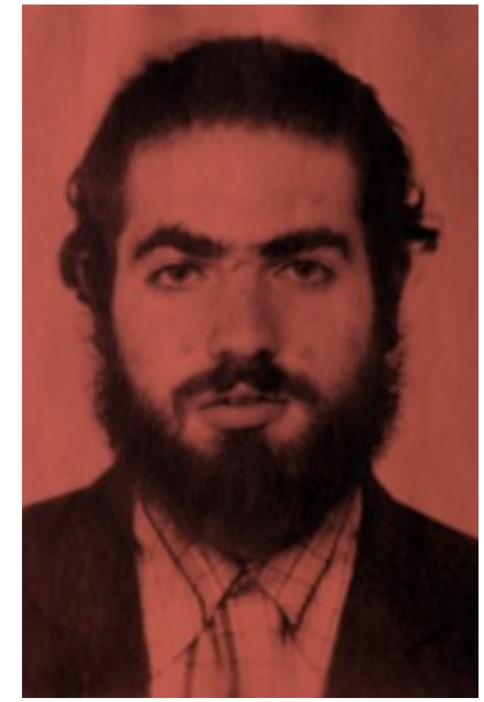

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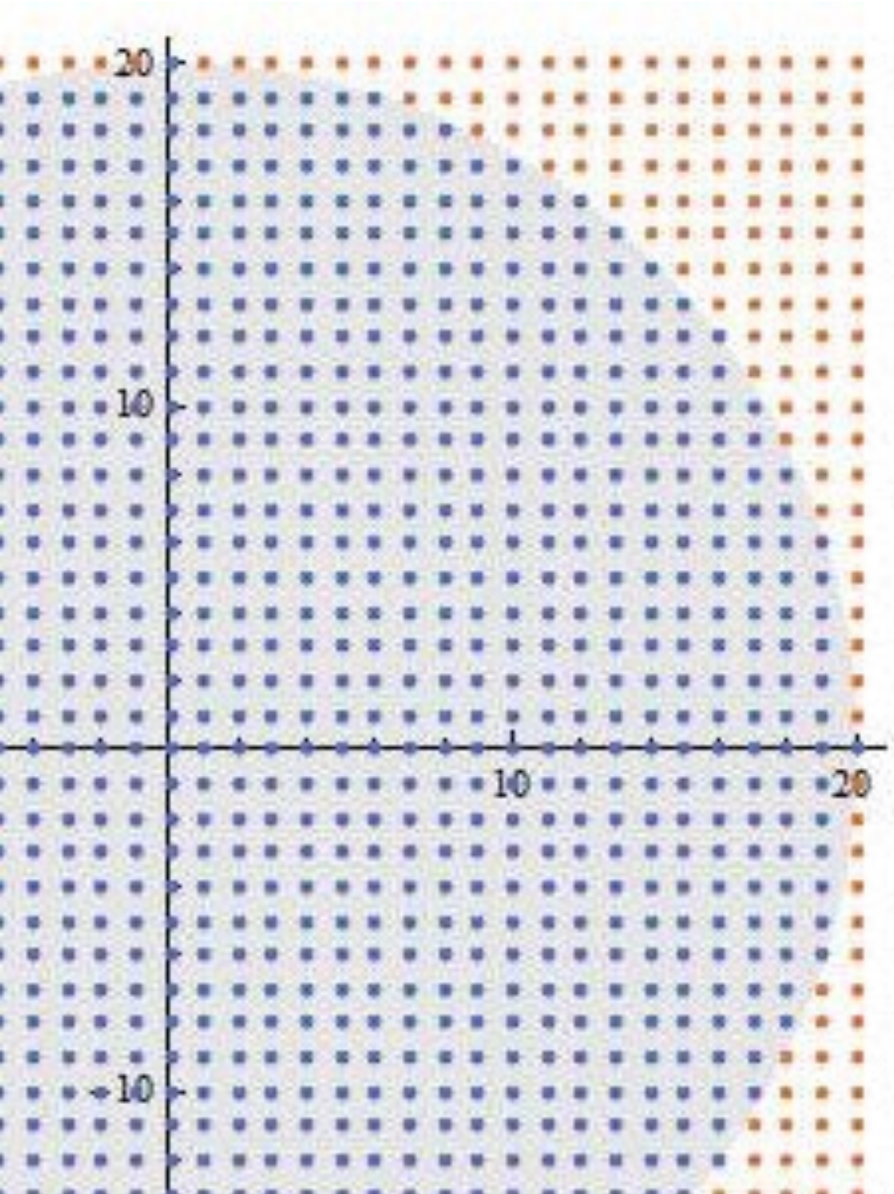

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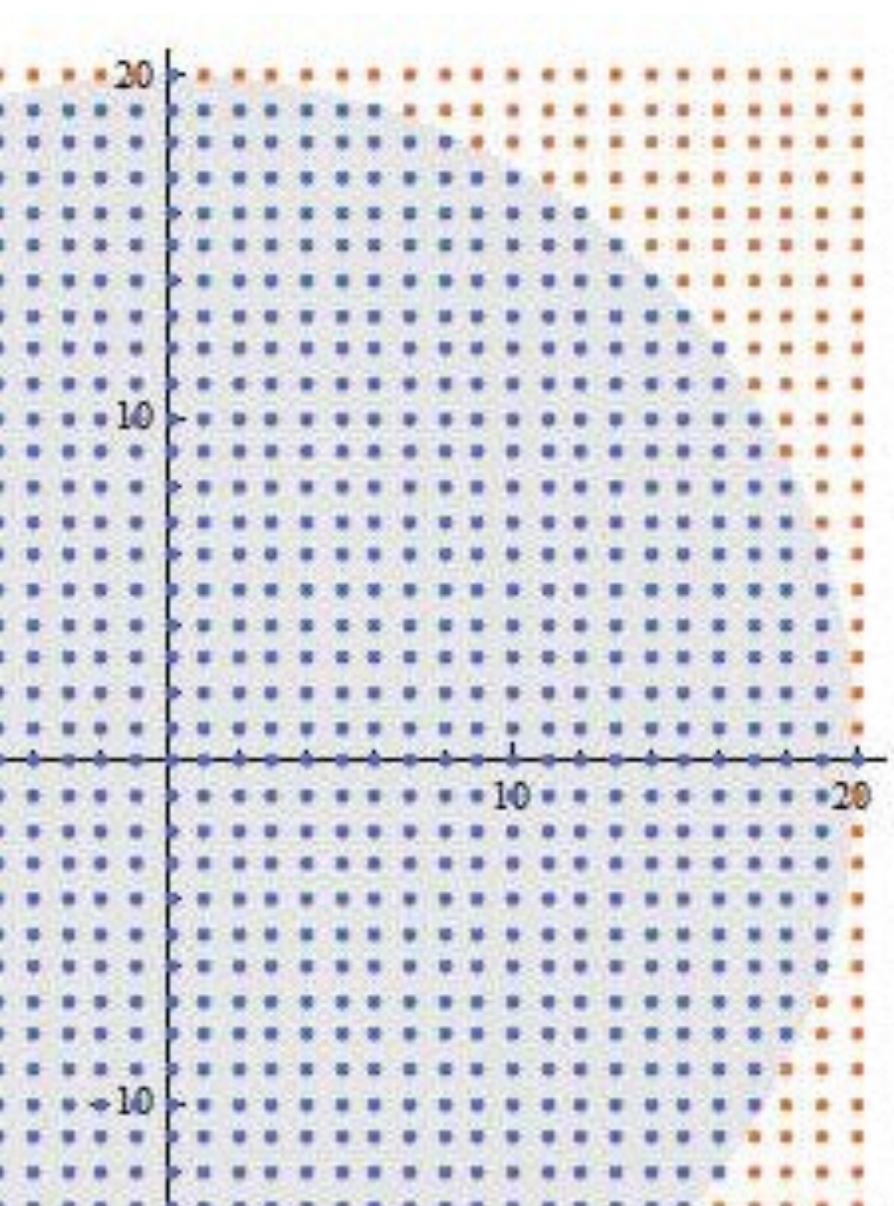

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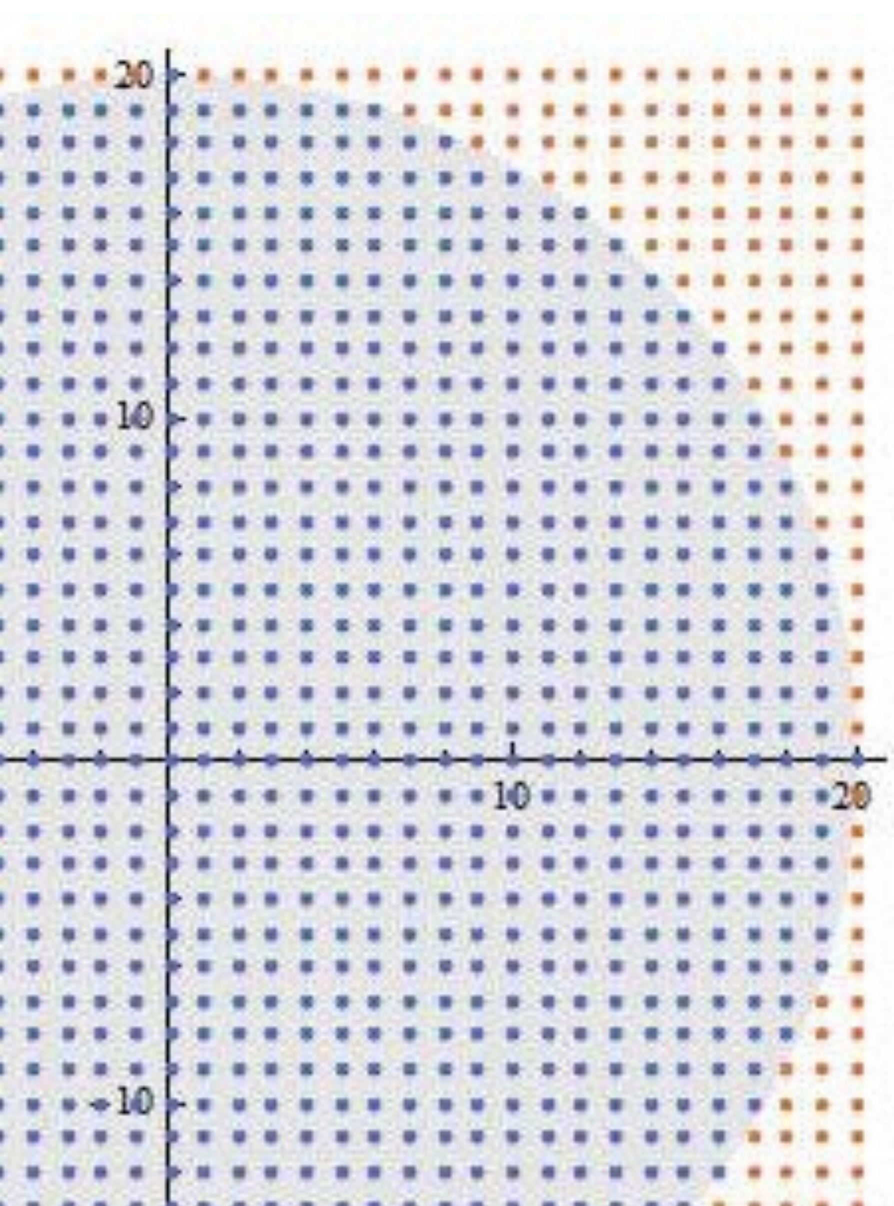

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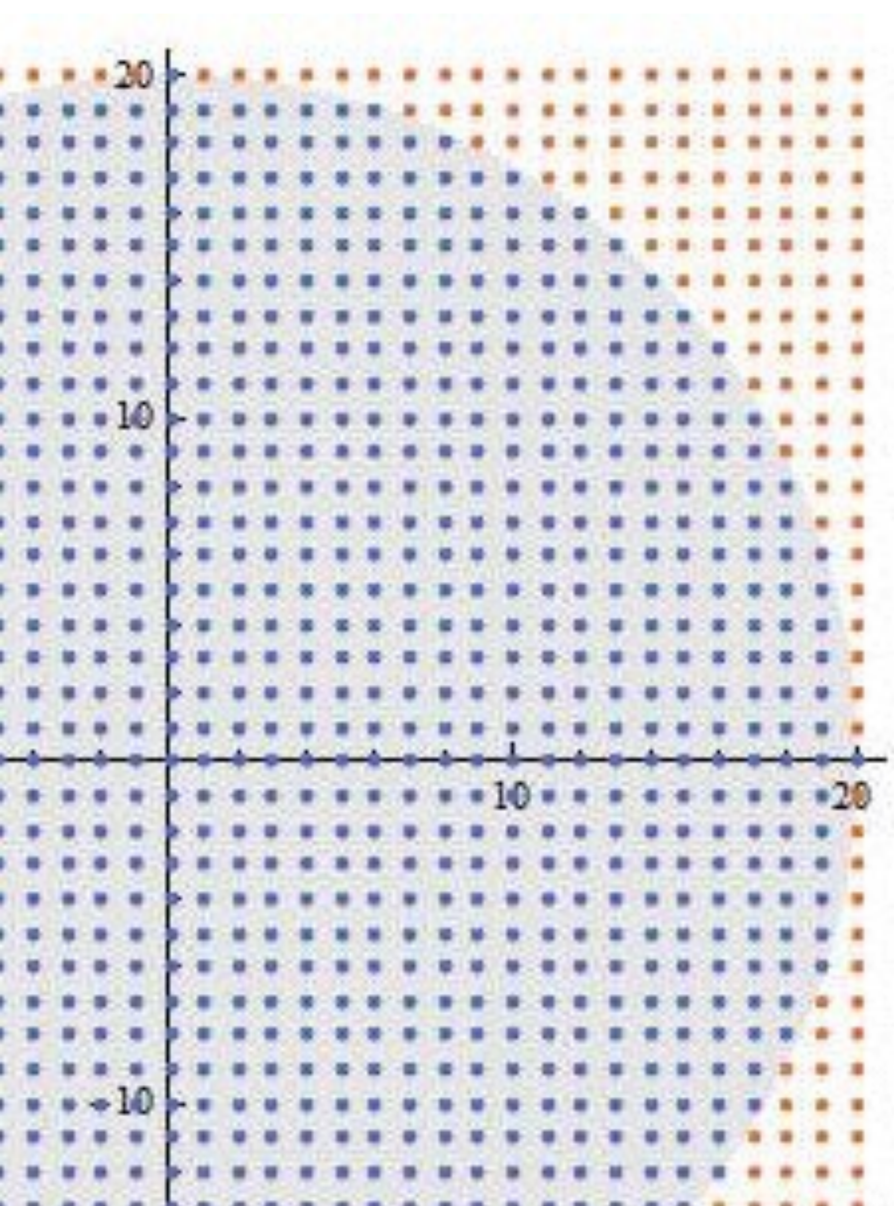

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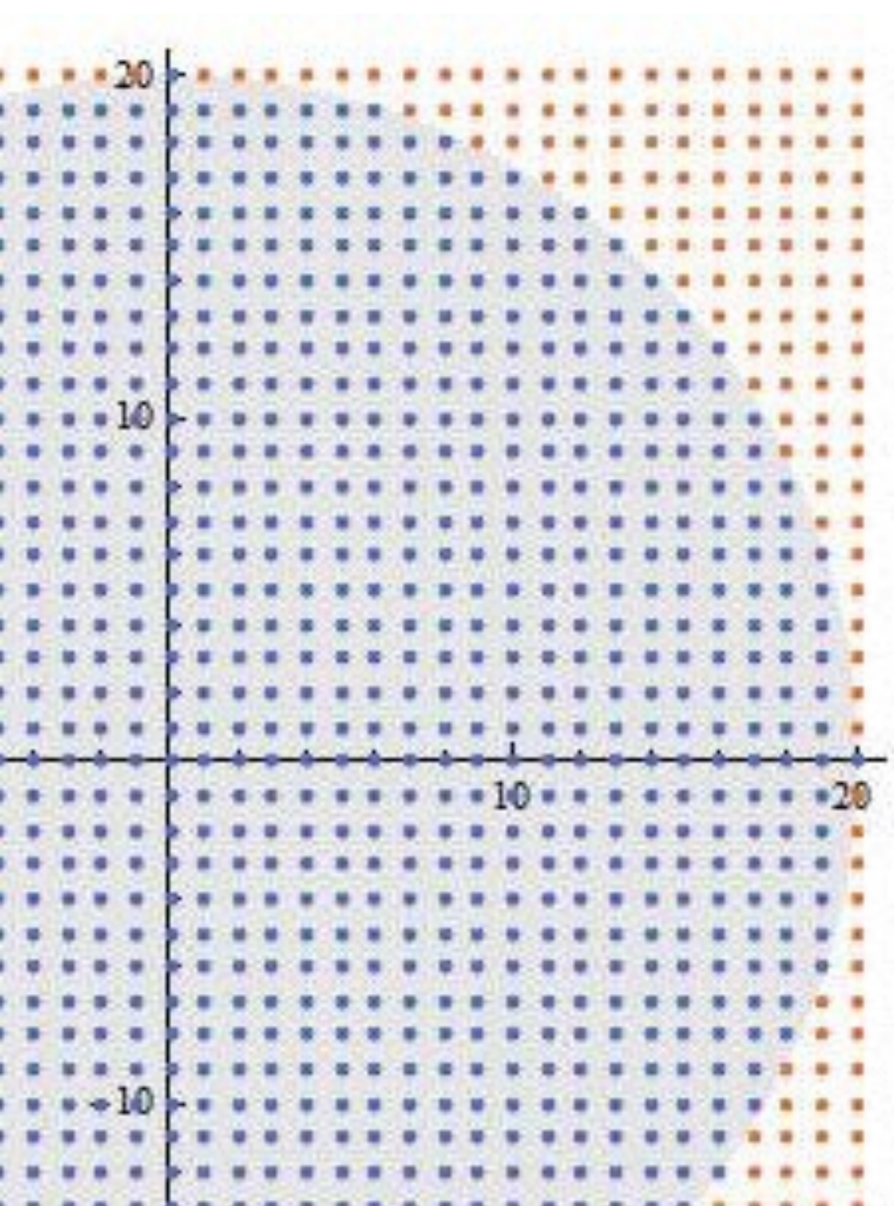

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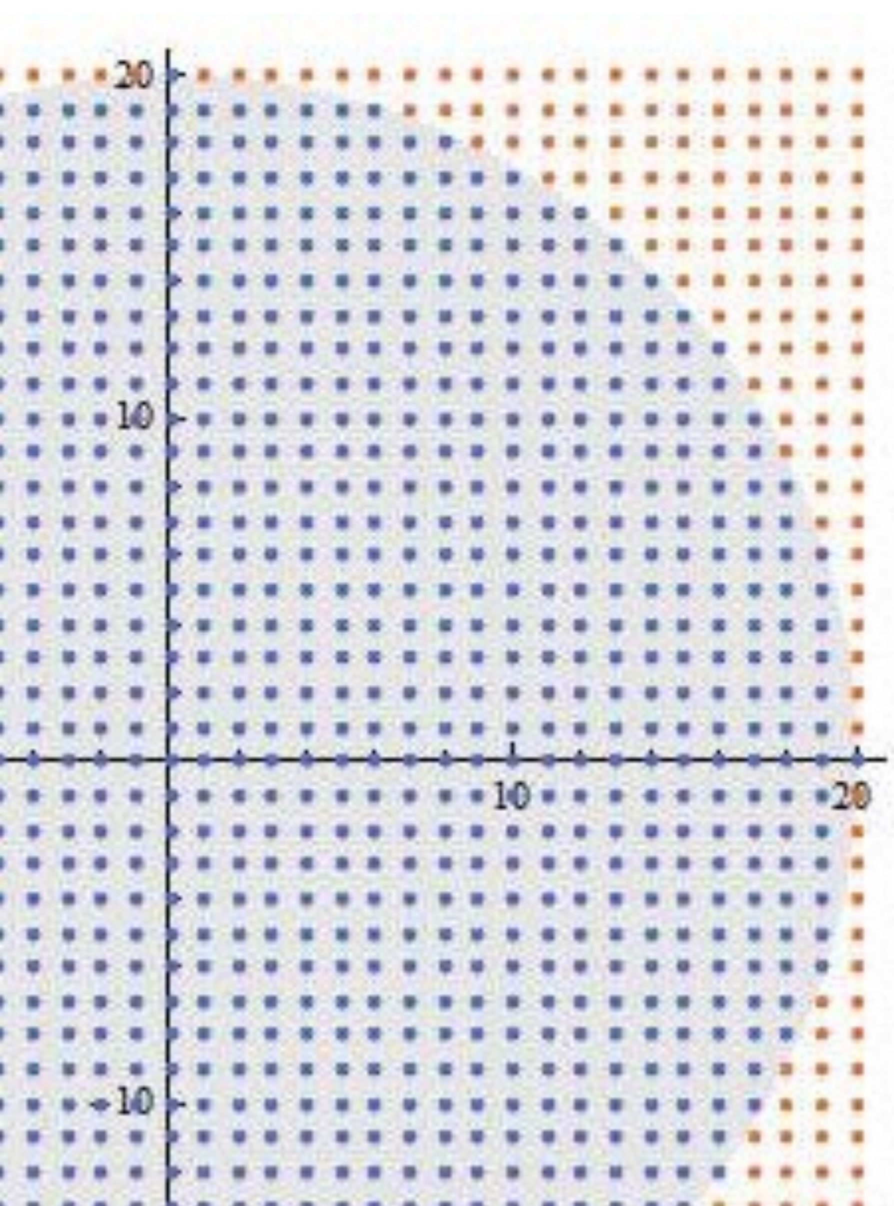

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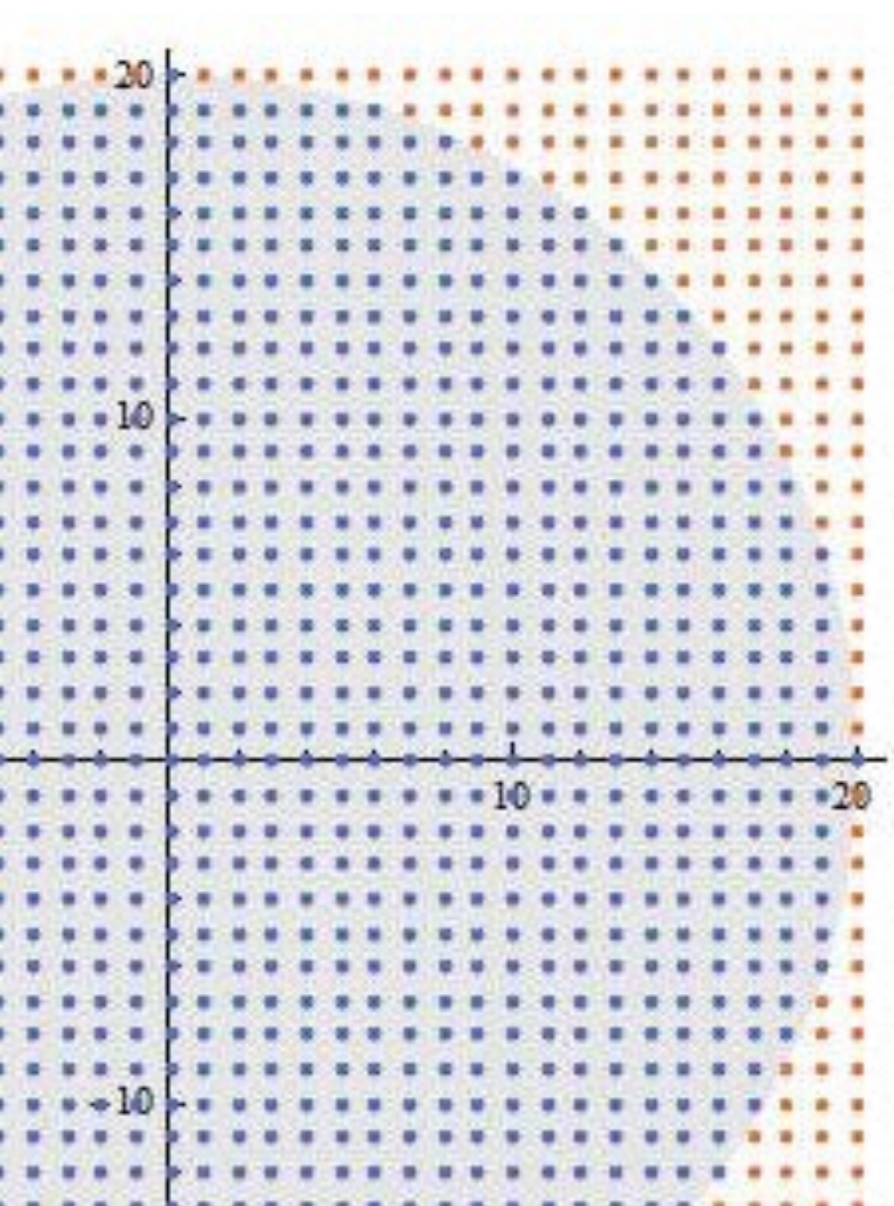

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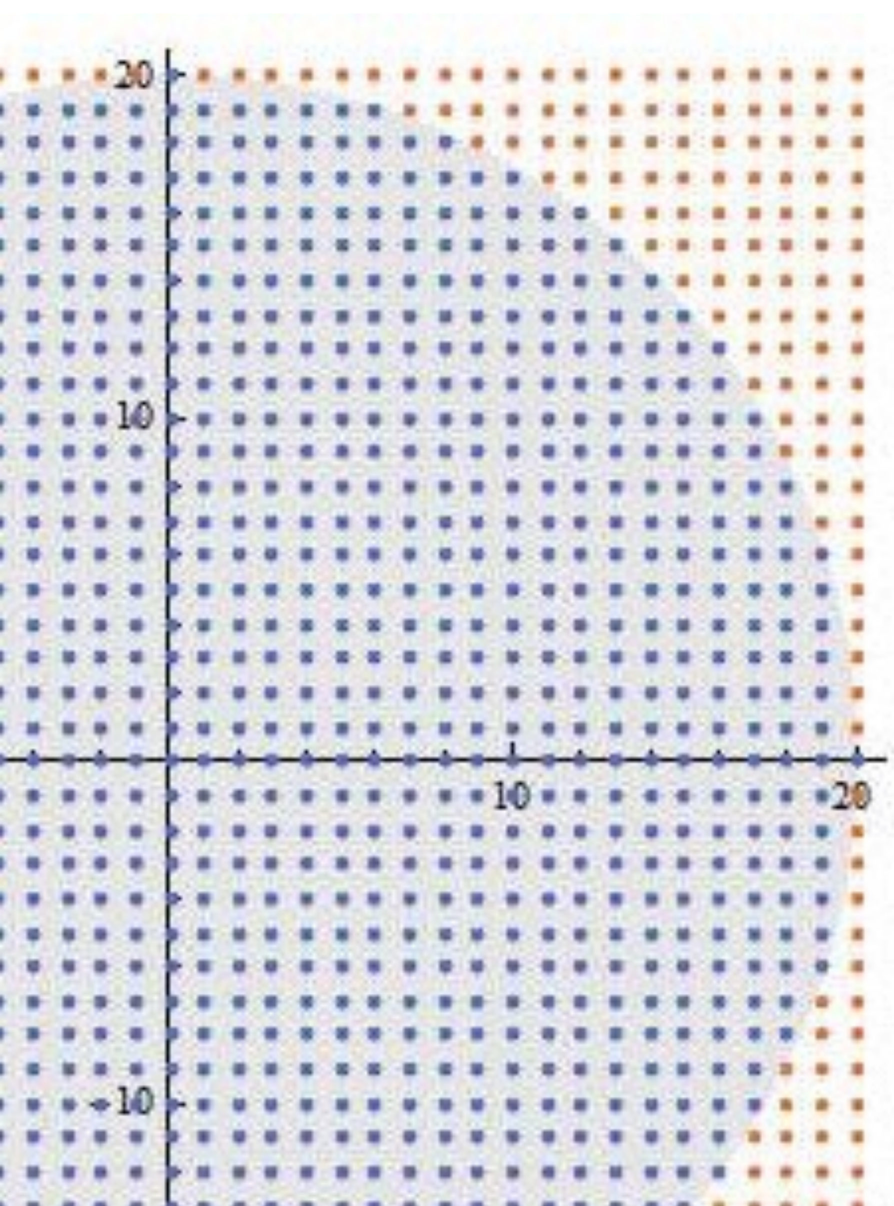

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- Sep 2014—This paper has been withdrawn as the authors have not succeeded producing an error free version

MORALLY CORRECT: TARSKI´S PROBLEMS

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A REPORT ON TARSKI'S DECIDABILITY PROBLEM: "ELEMENTARY THEORY OF FREE NONABELIAN GROUPS" BY O. KHARLAMPOVICH AND A. MYASNIKOV

Z. Sela

This paper contains a list of crucial mistakes and counterexamples to some of the main statements in the paper "Elementary theory of free nonabelian groups" by O. Kharlampovich and A. Myasnikov, which was published in Journal of Algebra in June 2006.

O. Kharlampovich and A. Myasnikov announced a solution to Tarski's problems on the elementary theory of the free groups in June 1998. Their work appears in a sequence of papers ending with the paper "Elementary theory of free nonabelian groups" that was published in the Journal of Algebra in 2006 [KM4]. I had already written a report on this paper, reviewing the published version as well as approximately 30 versions that preceded it, including serious mistakes that appeared in essential points in all the versions. In the current report I single out only the (fatal) mistakes in the published paper, together with counterexamples to many of its main statements.

The report starts with a short introduction that describes briefly the general approach to Tarski's problems that is presented in [Se1]-[Se7] and was adapted by the authors. As [Se1]-[Se7] mainly prove quantifier elimination, and the arguments there are not effective, I further explain what needs to be proved, in order to construct an effective procedure that will prove the decidability of the elementary theory of free (or more generally all torsion-free hyperbolic) groups.

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The report ends with a detailed account of mistakes and gaps in the paper under review (this list is essentially contained in the previous report). The list is by no means complete, but it is sufficient in order to demonstrate and verify my arguments. Many of the indicated mistakes are followed by counterexamples to the corresponding wrong claims in the paper.

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On Tarski's Decidability Problem: Response to Sela's Report

Olga Kharlampovich, Alexei Myasnikov

June 3, 2014

Abstract

This note provides a brief guide to the current state of the literature on Tarski's problems with emphasis on features that distinguish the approach based on combinatorial and algorithmic group theory from the topological approach to Tarski's problem. We use this note to provide corrections to some typos and to address some misconceptions from the recent report by Z. Sela about the relations between the concepts and results in the approaches to the Tarski problems. We were forced to read Sela's papers to be able to address some of his comments, and found errors in his papers 6, 3 and 4 on Diophantine Geometry published in GAFA and Israel J. Math. which we mention in Section 4. His proceedings of the ICM 2002 paper also contains wrong Theorem 6 (to make it correct one has to change the definition of non-elementary hyperbolic ω-residually free towers to make them equivalent to our coordinate groups of regular NTQ systems.)

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This is a response to Sela's "report" [16] on our paper "Elementary theory of a free group" [4].

Sela claims that the paper contains 57 mistakes that invalidate our results. These claims in some cases address inaccuracies in formulations (1, 5-8, 11 etc.) that were clarified later in the text or in the subsequent literature ([7], [6]) or are misprints (2,3,12 etc.), some corrected later [6]; in many cases they stem from misunderstanding or deliberate misreading the conditions (9-10, 13-15 etc.).

We discuss all these points in the following section. We are sure that Sela may immediately come up with another list of "mistakes" of the same kind. This has already been going on for about 10 years and has very little to do with mathematics.

In Section 3 we discuss the state of the theory of Algebraic Geometry in free groups from our point of view.

Finally we mention some serious inaccuracies and "similarities" in Sela' s papers that we are aware of. Possibly they can be fixed. We just want to say that "those who live in glass houses should not throw stones".

1

MORALLY CORRECT: TARSKI'S PROBLEMS

arXiv:1401.5711v1 [math.GR] 22 Jan 2014

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arXiv:1402.0482v3 [math.GR] 31 May 2014

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1

HOW MARGINAL ARE THESE CASES?

- Margulis likes to say “Every published paper is wrong.”

- Contested proofs
  - Foundations of symplectic geometry
  - Amenability of Thompson’s group
  - Hanna Neumann Conjecture

    ⋮

  - My last paper

CONCLUSION: THE ROLE OF PROOF NEEDS STUDY

- If we want to take seriously an investigation of validation, we should study
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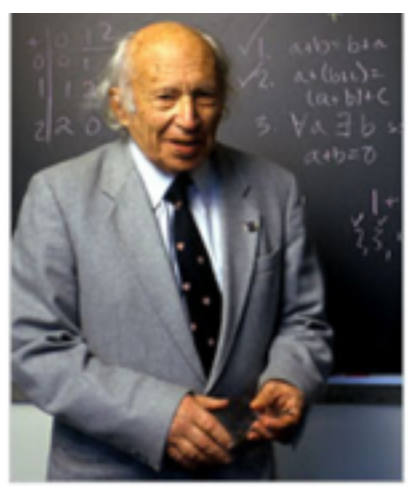

Courtesy of Rutgers University

Israel M. Gelfand as a professor of mathematics at Rutgers.

Dr. Gelfand did not achieve fame from attacking and solving famous, intractable problems. Instead, he was a pioneer in untrodden mathematical fields, laying the foundation and creating tools for others to use.

“People always compare him with great mathematicians like Euler or Hilbert or Poincaré,” said Vladimir Retakh, a professor of mathematics at Rutgers, where Dr. Gelfand spent most of his time as a visiting professor after leaving the Soviet Union in 1989.

Dr. Retakh said Vladimir Arnold, a prominent Russian mathematician, had contrasted the approaches of the Soviet Union’s two most famous mathematicians — Dr. Gelfand and Andrei Kolmogorov, who was Dr. Gelfand’s thesis adviser — with a travel analogy.

“Suppose they both arrived in a country with a lot of mountains,” Dr. Retakh said of Dr. Arnold’s comparison. “Kolmogorov would immediately try to climb the highest mountain. Gelfand would immediately start to build roads.”