Chapter 0

Introduction

MOON DUCHIN

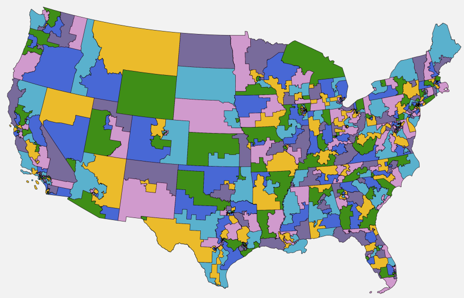
You've probably been hearing about gerrymandering lately. As I write in 2021, the U.S. Supreme Court has heard cases from Wisconsin, Maryland, Texas, Virginia, and North Carolina in its last two terms, if only to give back equivocal answers. Meanwhile, in Pennsylvania, plaintiffs pressed a partisan gerrymandering suit and prevailed in state court in 2018, followed by a frenzied few weeks with new proposed maps flying around, and finally a brand-new set of congressional districts, with sweeping effects for the Congressional delegation.[1] In North Carolina, multiple state-level cases fared better than the federal one, and the state put new congressional and legislative districts in place by the 2020 election, one tick before they would have had to be redrawn anyway in the new Census cycle.

What was at stake in these cases? Apparently quite a few seats, for one thing. Pennsylvania's new map coincided with a major shift in its congressional delegation, from 13–5 Republican control to a 9–9 split. Was that responsive to shifts in the vote, or a mere function of the carefully re-drawn lines?

Gerrymandering, or agenda-driven line-drawing, is a practice (and an anxiety) as old as the Republic. In a country that vests power in elected representatives, there will always be skirmishes and scrapping for control of the process, and in a system like our House of Representatives where winner-takes-all within a geographical district, the delineation itself is a natural battleground.

[1] Full disclosure: I got a front-seat view of Pennsylvania's districting reboot as a consulting expert for Governor Tom Wolf.

0.1 U.S. ELECTORAL DISTRICTS 101



- The U.S. House of Representatives has 435 members, ever since 1911. They must be elected one per district by common practice that was made official in law in the 1960s.[a] The picture shows the 432 in the continental U.S. circa 2019.
- State legislatures currently have 7308 members, all elected from districts. (Over 1000 of them are elected from multi-member districts.)
- Many thousands more elected representatives sit on city councils that are elected from districts—New York and Chicago have the two largest city councils, with 51 and 50 districts respectively.
- And then there are school districts, county commissions, ambulance districts, water boards, executive commissions, and more. The U.S. Census Bureau conducted a count of local governments in 2017, enumerating around 90,000 across the country.[b]
- "One person, one vote" jurisprudence from the Supreme Court (from the 1960s onward) tells us that electoral districts should be population-balanced within their jurisdiction—so zones to elect a school board must have nearly the same population, even if zones defining school attendance need not.
- We usually use *plurality* or "first past the post" voting in districts—i.e., the single candidate with the most votes wins. There are exceptions, like the multi-member legislative districts mentioned above, and the many local elections that use "at-large" schemes to fill several seats at once.
- We have two major parties, but the parties have shifted significantly over American history and certainly might continue to do so. For elections where candidates run with a party ID, there is often a primary several months in advance to pick the nominee in each party before the inter-party competition in the general election.
- Incumbency advantage is enormous. U.S. House races happen every two years, and for instance in 2016 only five incumbents ran for re-election but lost in the primary; another eight lost in the general election; and 380 were re-elected, for an overall success rate in the neighborhood of 97%.[c]

[a] Apportionment Act of 1911, Pub.L. 62–5; 2 U.S.C. §2c "no district to elect more than one Representative"

[b] `https://www.census.gov/programs-surveys/cog.html`

[c] Brookings Institute, *Vital Statistics on Congress*.

I am a mathematician with a background in geometry (shape) and dynamics (systems in motion). I have a long-standing investment in civil rights work and social movements. I'm also invested in social studies of science, and I like to think about how scientific argument circulates in politics, policy, and law—how technical expertise acts in the social sphere. This comes with a healthy skepticism of scores and metrics that promise to take a *complicated* thing and make it *simple*. So gerrymandering is an irresistible problem for me. It's all about peeling back layers of intuition about shapes, numbers, and power.

Today the primary image of gerrymandering centers on party politics, but the long history of manipulative redistricting has been driven by many other agendas, like back-room deals to make safe seats for incumbents or to dice up a district to stick it to a hated rival. And it's impossible to understand the current context or the bulk of the jurisprudence without contending with the history of schemes to suppress the political power of racial minorities, especially Black and Latino voters—not a practice of the past, but one that's even arguably on the rise in places where new demographic formations are visible. All of these flavors of gerrymandering have in common their basic structure: draw the lines to arrange pluralities for one set of voters and dilute the influence of the other voters.

This is stubbornly difficult to identify. People think they know gerrymandering by two hallmarks: bizarre shapes and disproportionate outcomes. But neither one is reliable.

1 HOW (NOT) TO SPOT A GERRYMANDER

1.1 BIZARRE SHAPES

We think crazy shapes tip us off to moustache-twirling gerrymanderers for a few reasons. The simplest is that we can easily imagine that the district line had to veer around wildly to include *this* pocket of people, but not *that* one. This seems especially likely if a district has been made to narrowly favor one party's voters in election after election. Another reason—if we expect that different kinds of people with shared community interests tend to clump together—is that jagged lines may indicate that an unspoken agenda has dominated over the contours of neighborhoods and communities. Finally and possibly most persuasively, we may worry that those who draw the lines just have too much detailed control over outcomes. Wildly winding boundaries flaunt the power of the pen.

The 1812 episode that gave us the word "gerrymander" sprang from this same pile of intuitions. The name is derived from Elbridge Gerry, governor of Massachusetts at the time. Gerry has quite a Founding Father pedigree—member of Congress, James Madison's vice president, a major player at the U.S. Constitutional Convention—so it's remarkable that he's mainly remembered in connection with nefarious redistricting. The "Gerry-mander," or Gerry's salamander, was the curvy state Senate district in Boston's North Shore that was allegedly drawn to favor one party, Gerry's Democratic-Republicans, over the rival Federalists (see Figure 1). A woodcut political cartoon ran in the Boston Gazette in 1812 with wings and claws and fangs suggestively added to the contours of the district to heighten its appearance of reptilian contortions—Figure 2 shows a Salem Gazette adaptation the next year.

So the idea that eccentric shapes are red flags for wrongdoing is old. And just as old is the idea that close-knit districts promote democratic ideals. Even before the notorious Gerry-mander, James Madison had written in the Federalist Papers (1787) that "the natural limit of a democracy is that distance from the central point which will just permit the most remote citizens to assemble as often as their public functions demand"—in other words, districts should be *transitable* to promote the possibility of deliberation. The new federalist model would knit these together: the United States was to be a republic built from these districts, serving as its constituent democracies. Forming districts of manageable size would ensure that the representatives have knowledge of "peculiar local interests" to be conveyed to the legislature (Fed. 14, 37, 56). So here, shape is in the mix but only as a correlate of function. In 1901, a federal apportionment act marked the first appearance in U.S. law of the vague desideratum that districts should be composed of "compact territory." That word *compact* then proliferated throughout the legal landscape of redistricting as a districting criterion, but almost always without a definition.[2]

Figure 1: Democratic-Republican Thomas Jefferson (left) and Federalist Alexander Hamilton (right) disagreeing over the compactness of this district. (Reenactment.)

Going back to the 1810s, the language from the Original Gerrymander broadsides is instructive. In the Salem Gazette, the *democratic* sins of the district are that it "cut up and severed this Commonwealth" much like "the arbitrary deeds of Napoleon when he partitioned the territories of innocent nations to suit his sovereign will." The *geographic* sins are those of its "peculiarities" of shape: three towns too far north, a town from a separate county "tacked on," and so on.

[2]Apportionment Act of 1901, 31 Statute 733. For a precise definition, see Chapter 1.

SALEM GAZETTE.

VOLUME XXVII.] SALEM, FRIDAY, APRIL 2, 1813. [No. 27.

THE GERRY-MANDER,
OR
ESSEX SOUTH DISTRICT FORMED INTO A MONSTER!

FEDERALISTS! FOLLOWERS OF WASHINGTON!

AGAIN behold and shudder at the exhibition of this terrific Dragon, brought forth to swallow and devour your Liberties and *equal* Rights. Unholy party spirit and inordinate love of power gave it birth;—your patriotism and hatred of tyranny must by one vigorous struggle strangle it in its infancy. The iniquitous Law, which cut up and severed this Commonwealth into Districts, is kindred to the arbitrary deeds of NAPOLEON when he partitioned the territories of innocent nations to suit his sovereign will. This Law inflicted a grievous wound on the Constitution,---it in fact subverts and *changes our Form of Government*, which ceases to be *Republican* as long as an *Aristocratic* HOUSE OF LORDS under the form of a Senate tyrannizes over the People, and silences and stifles the voice of the *Majority*.

When Tyranny and arbitrary Power thus make inroads upon the Rights of the People, what becomes the duty of the citizen? Shall he submit quietly and ignominiously to the decrees of Usurpers? Are the citizens of this Republic less jealous of their rights than their ancestors? Will you, then, permit a Party to disfranchise the People,---to convert the Senate Chamber into a Fortress in which ambitious office-seekers may entrench themselves and set at defiance the frowns of the People? No,---this usurping Faction must be dislodged from its strong-hold.

Arise, then, *injured Citizens!* Turn out! turn out! let Monday Next be the day of your Emancipation---by one manful Struggle reclaim your usurped Rights---and frown into obscurity those audacious men who unblushingly boasted---"*We have secured the Senate for ten years, and should have been Fools if we had not done it.*" Prove on election day that the Folly of their men is equal to their want of honesty and contempt of the People. Elect patriots who will be loyal to the Constitution, and faithful to the interests of the State,---men who will harmonize with our illustrious STRONG,---and who will not sell our Drafted Soldiery, like Hessians, to any Foreign Power.

requiſition their utmoſt cunning to deviſe means expreſsly to accommodate their own views and to promote their own intereſt, ſhould ſtill be unwilling to adhere to the policy preſcribed by themſelves.

Such a man would have to learn, that violence and ſubordination muſt be employed to accompliſh what diſhoneſty and trick had failed to ſecure. It is in fact the ſettled maxim of ſome, who, if the teſtimony of their own profeſſions is to be credited, have much love for the people and a ſacred regard for their rights, to contemn all legal proviſions, which impede their ambitious deſigns, however juſt and neceſſary to ſecure the peace and order of ſociety; to execute by force whatever they are prohibited to do by law, and to aggrandize themſelves without reſpect to juſtice or decency. Upon no other principle can we account for the conduct and activity of thoſe, who, at a late election in this town, ſcrupled not to bring to the poll, not only alien reſidents, who were known never to have been naturalized, not only a certain clergyman (the learned Commentator of Politics for this vicinity) whom they had, in common with the reſt of his brethren, diſqualified to vote in the choice of town officers, and thus to involve him in the turpitude of their tranſgreſſions, and to multiply the cauſes of repentance, (unleſs he ſo far believes, "that the end would ſanctify the means," that there would be *non cauſa pœnitentiæ*); but even to ſeduce citizens from *other* towns, to inveſt the minority with a power, which they would be ſure to abuſe. He, who promiſes freedom to the captive, when he increaſes the number and weight of his chains, acts as confidently, as he does, who declaims much and loud about the liberties of the people and the rights of his country, when all his actions evince, that he regards neither the one nor the other, any farther, than he diſcovers a direct advancement of his own intereſt. It is greatly to be deſired, that *honeſt* men would ſometimes ſuffer themſelves to be influenced as well by objects of *viſion*, as *hearing*, and compare *actions* with *profeſſions*; and then let judgment decide upon the worth of character, and the ſafety of repoſing confidence. CIVIS.

MR. PRINTER,—I am a native of *Great-Britain*, and ſome years ago was naturalized at *Gottenburg*; but for ſeveral months I have been engaged in trade in this State. I have ſeen very preſſing notices of the Marſhal for alien enemies to report themſelves forthwith While I remain here I wiſh to comply with the laws and ordinances of the country, and feel no enmity towards the government or people. A peace or change of times would deſtroy the profits of my preſent ſpeculations, and in that caſe I ſhould probably ſoon depart. Moreover, I do not find that the U. S. of America are at war with my adopted country. Now I will thauk any of your correſpondents to inform we what is proper or expected from one in my circumſtances; as there are many in the country, and more expected, ſimilarly ſituated. A. B.

"UNDER A SIGNATURE,"

Is the correct, and was once the universally received phrase. But a spirit of innovation and hyper-criticism directed by gross misconception has engendered and brought into too general use the awkward and absurd term, "*over a signature.*"—To show how mischievous tolerated abuses are, we would refer to the following from the Boston Gazette. "*The Road to Peace.*

"Messrs. Russell & Cutler—I have read a very sensible, but small tract [compose your muscles, gentle reader] *over the above title*," &c.

Now, the tract, "*over*" this title is worthy of all praise for its correctness and candor, and may be read *without offence* and certainly with advantage, *by all parties*. It is peculiarly calculated for *distribution*—it *repels no reader*, whatever be his politics or character.

Given *over* my hand and seal, March, 1813.

THE GOOD OLD WAY.

DARKNESS VISIBLE.

Mons. GALLATINI has directed the Lights near the Chesapeake and Nantucket to be extinguished, so that our Enemies may be left in the dark; and of course it will be perfectly easy to place Torpedoes under the Fleet and blow it out of our waters.——How perfectly in character is it for our present rulers to *extinguish* instead of illumining, and to scatter *darkness* where their predecessors diffused *light*. It is the sport of these Proclamation-men, like Mad Tom,

"To play at bowls with sun and moon,
"And shade them in eclipses."

It is expected that every man who does not wiſh to be drafted as a conſcript and ſent out of the State to fight the battles of the Southern Nabobs—every Farmer who wiſhes to prevent his farm and ſtock from being ſeized and ſold to pay the enormous taxes which are ſhortly to be laid on them, will turn out on the firſt Monday of April next, end ſupport the Peace Ticket.—New Hampſhire has ſet a bright example; let us follow it. You all know you were proſperous and happy under Waſhington; you are now wretched and miſerable under Madiſon.—You cannot, you will not give your ſuffrages for men who are in favor of the preſent war, unleſs you are unjuſt to yourſelves, and unfaithful to your country.—*Auguſta Herald.*

Salem,
FRIDAY, APRIL 2, 1813.

SOUTH with a Vengeance!

The Senatorial Diſtrict made and called "ESSEX SOUTH *Diſtrict*," by a Gerrymander Legiſlature, has the following *peculiarities*:

1. It contains three Towns (Saliſbury, Ameſbury and Haverhill) more *northerly* than any other town in the County.

2. An Eaſt and Weſt line drawn through the centre of the County leaves Saliſbury, Ameſbury, Haverhill, Methuen, a large part of Andover and a ſmall part of Middleton [ſix] on the *Northern* ſide.

3. The only towns wholly in the *ſouthern* ſection of the county that the diſtrict contains, are Danvers, Lynnfield, Salem, Marblehead and Lynn [five.]

4. The *northernmoſt* town of *another county* is tacked on it.

☞ There neighbor, take a piece of chalk and ſee if it does not come out ſo.

Meaſures for Defence have been adopted in all the ports threatened by the Britiſh Squadron. At Wilmington a Committee of Safety has been appointed. The Honorable Meſſrs. BAYARD and HORSEY, (U. S. Senators from Delaware) and other zealous Federaliſts conſtitute this Committee. Theſe are the Patriots on whom the country muſt rely when the hour of extreme peril approaches. Our democratic rulers have ſufficient talents to plunge the nation in miſery and calamity, but none but Federaliſts can reſcue the nation and ſave its "drowning Honor." The deceaſed Heroes WASHINGTON, LINCOLN, KNOX, WAYNE, MORGAN and HAMILTON,—the Soldiers of Liberty who atchieved our Independence,—were zealous and decided *Federaliſts* to the laſt hour of their lives;—and even WASHINGTON was calumniated and traduced as a Britiſh Tory by the Democratic party,—which party he ſolemnly declared in a letter to a friend was "A CURSE TO THIS COUNTRY." This ſame party is now ſcourging the country with War, and the People *feel* the truth of the words of WASHINGTON.

The Right of Suffrage.

ALL men will agree in the importance of having the right of ſuffrage properly defined by law; and moſt men, when uninfluenced by paſſion, will acknowledge the neceſſity of enforcing the law with the utmoſt ſtrictneſs, and can correctly calculate the immenſe benefits which would reſult from ſuch a courſe.—Theſe are the natural dictates of the underſtanding and of conſcience. But man in *theory* and in *action* is not the ſame being. In the diſpaſſionate moments of ſpeculation, he liſtens to the ſuggeſtions of his reaſon, and approves of the admonitions of his underſtanding; but in practice he acknowledges no law but convenience, and feels no ſatisfaction for his exertions except in triumph and victory.—To a perſon, whoſe aſtoniſhment had not been ſo often excited, till he had become incapable of new ſurpriſe, by the moral phenomena which are continually occurring in the political world in theſe days of ſtrange deluſion and of diſaſters; it would appear incredible, that a party of men, after they had had the opportunity of putting in

Figure 2: The O.G. (Original Gerrymander)

There's no question that the outline of a crooked reptile is doing a lot of work on your intuition. If this feels like an "eyeball test," that's exactly what it is, and it's a major driver in redistricting to this day. Thirty-seven states have some sort of shape regulation on the books, and in almost every case (at least until the map goes to court!) the eyeball test is king.

But the problem is that the eyeballed outline of a district tells you a very partial, and often very misleading, story. Take Alabama's 1st district (Figure 3), bounded to the south by the jagged Gulf coast and to the north by a pair of rivers.

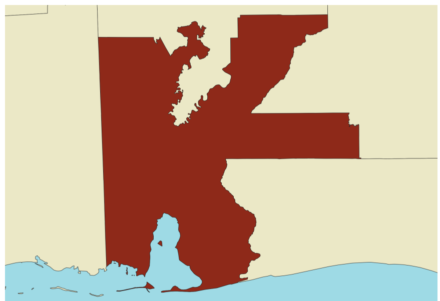


Figure 3: Alabama's 1st district.

The parts of its boundary that are not dictated by physical geography tend to follow county lines fairly faithfully. And county lines may be tortuous themselves, but you wouldn't want to punish a district for following them! (In fact, many states have rules telling you that district lines should follow county lines "to the extent practicable.") This spotlights a balancing act that is both real and often scapegoated: districters quite often will claim that other districting rules and principles forced horrible shapes on them. The plausibility of this claim.... varies, to say the least.

For instance, Figure 4 shows a pair of notorious districts, one from North Carolina and one from Maryland. Party politicians on both sides of the aisle claimed (and may have believed!) that the shape of NC-12 was forced on them by the Voting Rights Act. And at least one leading political figure asserted (but surely did not believe) that MD-3 had to look this tortured to hit a precise population number.[3]

Certainly there can be benign reasons for ugly shapes. Even more importantly, districts that are plump and squat and symmetrical to the eye offer no real seal of quality. For example, in the 2018 redistricting scrum in Pennsylvania, the state Supreme Court invalidated the 2011 Congressional plan and asked for a new one. Interestingly, the court order named a number of metrics that had to be calculated for any new plan, including five somewhat different scores of shape to be reported for each district, without specifying what role all those numbers would have in their decision. First crack at a new plan goes to the legislature, which had the opportunity to commission a new plan and to pass it as a bill. They didn't end up voting on it, but the Republican leaders Mike Turzai (House) and Joe Scarnati

[3]John T. Willis, the Democratic party stalwart who chaired the redistricting subcommittee, said: "It's a very complex situation, and population is the No. 1 driving characteristic. ...All of our congressional districts don't deviate by more than one person." See E. Batista, "Politics Makes Strange Bedfellows, Even Stranger Congressional Boundaries," perma.cc/P6Z4-S2NL.

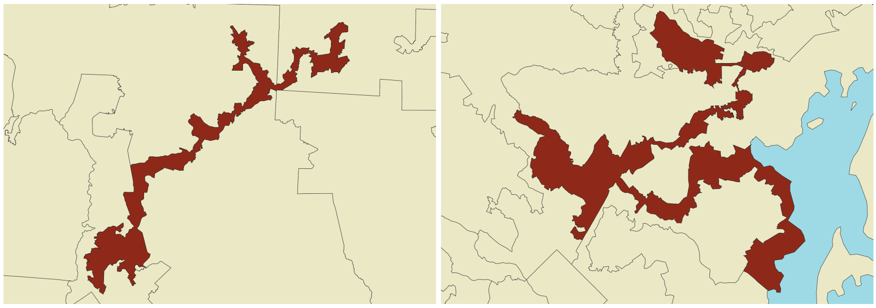


Figure 4: Examples of scapegoating: North Carolina's 12th district and Maryland's 3rd, circa 2013.

(Senate) floated an alternative plan on Twitter before filing it with the court—so I'll call it "the Twitter plan." The Twitter plan achieved glowing compactness scores, under all five formulas specified by the court, relative to the much-mocked 2011 enacted plan that it was aimed at replacing. But the court found that despite its more pleasing forms, the plan locked in the same extreme partisan skew as its predecessor. So in Pennsylvania, you can get extreme performance with innocent shapes (see Figure 5).

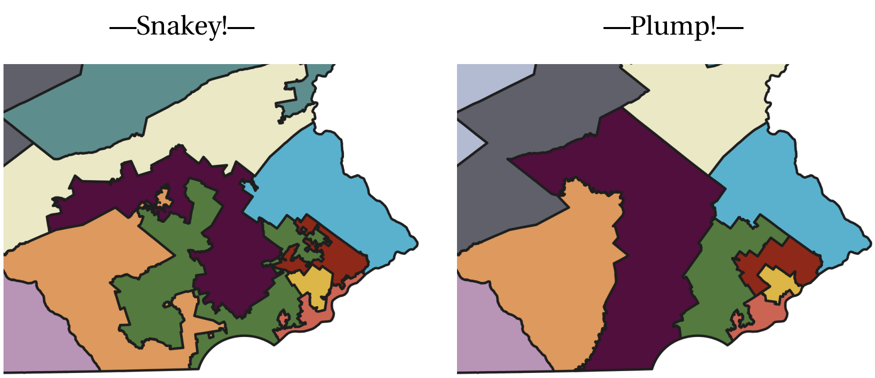


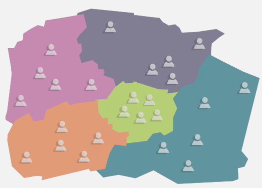
Figure 5: Philadelphia-area inset of the 2011 enacted Congressional plan (left) and the replacement map proposed in 2018 by Republican legislative leaders (the "Twitter plan," right). The replacement *looks* great, but both plans only have 4 out of 18 Democratic-majority districts when laid over the 2016 Senate vote pattern, which was nearly equal between the two major parties. (Compare Figure 12.)

In fact, some of the reason why shapes were often so flagrantly ugly in the past is not that horrible contours were strictly needed for more extreme partisan performance, but that the right kinds of pressure were not yet in force to rein them in.[4] What's more, the Twitter plan is not the exception—even strong shape imperatives may fail to constrain. Under scrutiny, line-drawers can often lock in all the advantage afforded by an ugly plan while keeping the shapes nice and plump.

[4] The era of shape-based legal invalidation really began in the 1990s with the so-called Shaw Line of Supreme Court cases (see Chapter 7), when the court grumbled about—but still began to engage in—"endless beauty contests" about district appearance.

0.2 THE RULES OF REDISTRICTING

Redistricting is made extremely complicated by a patchwork of rules that are typically unranked and often fuzzy to boot. Here's a quick primer on the "traditional redistricting principles" across the country, plus some that are less traditional but still make appearances.

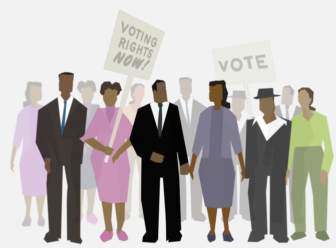


Equal population—*Districts within a polity should all have very close to the same population.* The standard way to count is to use the Decennial Census numbers, which is one of the reasons why the Census is so important. This rule applies to the whole nation, and these days any two Congressional districts within a state will most often have a zero- or one-person difference in their Census count![a]

Contiguous—*Each district should be a single connected component.* You may be surprised to hear that only around 30 states require this property by law. This rule is mostly straightforward except when you're building from units that are themselves disconnected, or where there are water crossings to consider.

Compact—*The districts should be reasonably shaped.* ...Whatever that means! Language varies on this one, but for the most part it's a matter of the eyeball test. At least 37 states reference this principle.

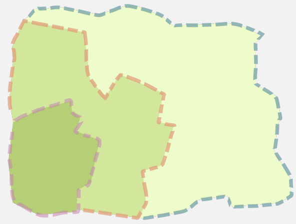
Voting Rights Act—*The districts must not undercut the opportunity for minority communities to elect candidates of choice.* This has been a federal law on the books since 1965 and has a formidable (and formidably complicated) legal history and practice.



Communities of interest—*Groups with significant shared interests should be strategically placed in order to boost their voice in government.* While it's one of the most conceptually important, this principle is especially open-ended. Shared interests could be about industry, environment, or culture, and groups are sometimes better served by being kept together and sometimes by forming a significant part of multiple districts. More states will take concrete steps toward COI consideration in the 2021 redistricting cycle than in any previous cycle.

Political boundaries—*Counties, cities, and other relevant jurisdictions should not be split among multiple districts when there is a way to keep them whole.* In some states, this is phrased as a preference that district lines should follow political boundary lines.

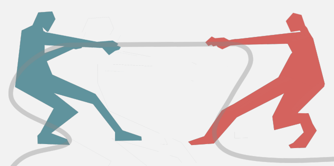
Units—Some states prescribe which building-block pieces plans should be assembled from. For instance, Louisiana and New Mexico mandate whole precincts in legislative plans, and Iowa requires that counties be kept whole in congressional districts.



Nesting—Eight states currently require the state House districts to nest inside the state Senate districts two-to-one, and two additional states require three-to-one nesting.

Incumbency—In some states, there is a rule on the books that implies that *new maps should avoid pairing incumbents to run against each other*. (Pairing incumbents also goes by the colorful name "double-bunking"!) In other states, the rules forbid having the redistricters consider incumbency at all.

Partisan properties—A handful of states have rules indicating that there is a priority on the creation of *competitive* districts or districts that react *responsively* to changes in voter opinion, and numerous states have considered adopting language of that kind. Several other states forbid considering partisan data in the redistricting process.



[a] ncsl.org/research/redistricting/2010-ncsl-redistricting-deviation-table.aspx

1.2 DISPROPORTIONATE OUTCOMES

So district shape will not do the trick on its own. How about if we cut out the middleman and get right to the bottom line, studying the extent to which the representatives match the electorate. Many people hold the strong intuition that disproportions give prima facie evidence of abuse. That is, a group with 30% of the votes *would have gotten* 30% of the seats, if the lines had not been rigged.

But not so fast. Let's zoom in on a particular case to understand some of the root causes of disproportionate outcomes. We'll look at a subgroup that reliably has over 1/3 of the votes but is locked out of even 1/9 of the Congressional representation: Republicans in my home state of Massachusetts.[5] This is a situation where even

[5] Let me flag at the outset that it's hard to directly measure people's party preferences for Congress

if your heart expects or desires a proportional outcome, the structure gods are cruel—it can't be done.

If you consider the elections for President and U.S. Senate held in MA since the year 2000, the Republican share of the statewide vote is most often between 30 and 40%, averaging over 36%. Since that's well over a third of the vote and we have nine seats to fill, you might expect a fair map to send three Republicans to the House in each cycle; meanwhile, the last time a Republican won *any* MA Congressional district was in 1994.[6] That is thirteen straight election cycles of total Republican lockout. So we must be looking at a vicious gerrymander that denies Republicans their rightful opportunity districts, right?

Except the mathematics here exonerates the Bay State line-drawers. The Bush-Gore election in 2000 is a great example. There is literally no way to put together a subset of the state's 351 towns making up enough population for a district—no matter now disconnected and scattered—that preferred Bush. That sounds like a paradox, but it's easily explained. Though Bush won 35.2% of the statewide vote, only 32 towns preferred Bush outright, making up under 3% of the state population. Preferences were very flat around the average, and there just aren't enough Bush-majority towns to anchor a district, no matter how cleverly you group them.[7]

The state started reporting more granular precinct-level results just after that, giving us an opportunity to see that the pattern held up in Massachusetts all through that Census cycle. Kenneth Chase, the Republican challenger to Ted Kennedy in 2006, cracked 30% of the statewide vote. But once again the districting numbers don't shake out for Chase voters. It is mathematically impossible to create a single district-sized grouping of *precincts* that preferred Chase; this is a realistic redistricting setting because precincts are typically preserved whole in Massachusetts legislative plans and rarely broken up in congressional plans. Chase voters simply were not clustered enough for a district to give them access to representation.

The problem is that even though Republican voters are nearly a third of the state, they are also about a third of every town and a third of every precinct—and a third of every household, as far as I know!—so no combination of units can combine to form a Republican majority, even if you throw niceties like compactness and contiguity to the winds. And this phenomenon carries over to any group in the numerical minority. You need a certain level of nonuniformity in the distribution for districting to offer even a theoretical opportunity to elect. The takeaway is that districts are ineffective if a minority is dispersed.

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from Massachusetts, because the races are so often uncontested, as five out of nine seats were in 2016. Also, like many states, Massachusetts votes Democratic for national office but loves its Republican governors. But really I just want to make a point about the consequences of certain distributions of *votes*, so we can look at statewide elections for federal office—Senate and President—to understand that.

[6] If there are nine districts, each has about 11% of the state population—since you just need a plurality to prevail, you should be able to control a district with just 6% of the statewide vote. (The apportionment for Massachusetts dropped from 10 to 9 during this timeframe, but I will stick with 9 districts to simplify the discussion.) So Republicans routinely get more than *six times* the vote support needed to control a district.

[7] Exercise for the enterprising reader: collecting units in order of Republican vote share is a less effective greedy strategy than going by Republican margin per capita. You can find an appropriate sorting lemma in Duchin et al. [1].

So as a system of representation, districting doesn't start out looking like it will provide strong guarantees for minority groups (which includes Republicans in Massachusetts). And that's only looking at the population shares in the units and not at their spatial arrangement, which often compounds the difficulty. Even the mildest constraints on shape, like requiring that each district be one connected piece, make it harder to convert scattered votes for a minority-preferred candidate into representation. In a fairly ironic turn, this means that minority groups with the most strongly segregated geographic patterns—like racial groups historically targeted by discriminatory housing policy—may be in the *best* position to leverage the system of districts to secure representation. Spatially dispersed groups have no hope. Suddenly it looks unreasonable to expect that representative democracy can make good use of winner-take-all districts.

We have identified a problem with the system: districts beget disproportion. Let's look to mathematics to measure the extent of this problem, and to try to understand some of the mechanisms that cause it.

2 THE UNIVERSE OF POSSIBILITIES

We want to understand how districts might be able to provide some minority representation within a majoritarian paradigm. More broadly, we just want to understand what they can and can't do. It would be enormously useful to be able to survey all of the possible districting plans that satisfy some basic constraints, and then reason from there.

Mathematicians like to ease into a hard problem by first abstracting to a "toy problem"—an ultra-simplified model that helps illuminate structural issues. So instead of directly tackling the question at hand (what are all the ways to divide the geographical units in a state into nearly equipopulous districts?) we'll start much simpler: redistricting a small square grid with homogeneous units. This is already hard, as it turns out.

2.1 NUMBER

Maybe I've made some progress in convincing you that neither weird shape nor glaring disproportion gives a sure stand-alone indicator of gerrymandering. If you want to evaluate whether an election result should make you distrust the districts, you should really be comparing the plan to *other possible ways of districting the same jurisdiction.* The catch is that studying the universe of possible plans becomes an intractable problem.

When you think about "big data," you probably think of space exploration and medical imaging. It may come as a surprise that the humble math problem of how many ways to cut up a square pie belongs in the same conversation.

Think of a simple 4×4 grid, and suppose you want to divide it into four "districts" of equal size, 4 units each. The only requirement is that the districts should be contiguous. If we imagine the regions on a chessboard and we interpret contiguity to mean that a rook (traveling vertically and horizontally) should be able to visit the whole district, then there are exactly 117 ways to do it, summarized in Figure 6.[8] I'll denote rook-contiguous partitions of the $n \times n$ grid into k equal districts as the $n \times n \to k$ problem, for short. A cleverly programmed laptop can generate the $4 \times 4 \to 4$ solutions instantly.

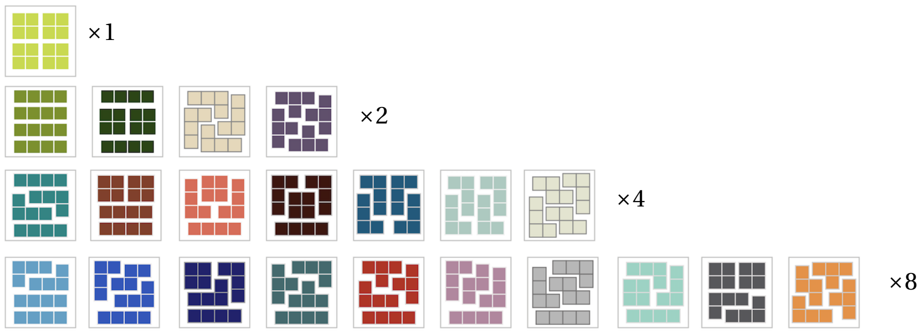


Figure 6: The 4×4 grid has 117 four-district plans—start with the 22 types shown here and apply rotations and reflections to get the full list. Try for yourself! No matter how you rotate or reflect the first plan, it looks the same (so it only contributes ×1 to the ultimate list), but each plan on the next row is one of a pair of variants (so they contribute ×2).

But to my surprise—forgive me, I'm trained as a theorist—I've learned that it's not obvious how to get even a high-performance machine running the best known algorithms to count all the possible configurations in a reasonable amount of time. At the time of writing, our best methods can handle $7 \times 7 \to 7$ in seconds and $8 \times 8 \to 8$ in minutes, but the 9×9 is a much more formidable computing task and the 10×10 is out of reach. Now try 18 districts built from Pennsylvania precincts!—it's not only a far bigger problem (9059 units) but has a more complicated connection topology of the units, with no symmetry to exploit. Forget about getting an answer during a 10-year census cycle; this complete enumeration calculation almost certainly can't be done before the heat death of the universe.

One reason for that is that the contiguity and balance constraints are stubbornly nonlocal, meaning that if you just look in one small neighborhood you can't be sure that a district is globally connected or that it's the right size. And these requirements have a lot of bite: unconstrained, there are roughly $4^{16}/4! = 179$ million ways to label 16 grid squares as belonging to district 1, 2, 3, or 4. Balance (insisting that each district is of equal size) cuts it down to $\binom{16}{4}\binom{12}{4}\binom{8}{4}/4! = 2.6$ million. Contiguity without balance cuts it down to 62,741. And both together leave you with just 117. So "brute force" algorithms that have to check all possible labelings just don't scale. This seems to call for a clever idea and not just the determination to search exhaustively.

Unfortunately, the problem doesn't reduce in a nice way: knowing the full answer for smaller grids gets us nowhere at all with the $n \times n$. (In math-speak, the problem lacks *recursive* structure.) So to find the very large number of valid partitions, you're searching blindly in an exorbitantly larger ambient space.

[8]If corner adjacency is permitted—so-called "queen contiguity"—the number jumps to 2620.

2.2 CLUSTERING

Since we can't simply build out all the plans, we will need to start understanding what *features* of the problem have important consequences for the measurements we care about. If we are trying to divide a population of two types into districts, it really matters how that population is laid out over the area we are dividing. Let's call this the *political geography*. We've already seen that political geography doomed the hapless Republican voters of Massachusetts—they were too uniformly distributed across the units (towns or precincts) to secure representation. They were not clustered enough.

On the other hand, conventional wisdom in redistricting carries the strong view that 21st century Democrats are disadvantaged by *excessive* clustering. "Democrats pack themselves!" as the slogan goes—because the votes are densely arranged in dense cities, even party-blind redistricting tends to create wastefully high Democratic percentages in urban districts, causing inefficient *packing* (shares far higher than needed) in parts of the map and *cracking* (shares just below the winning threshold) in others. But the math is actually subtle.

If too little clustering is bad and too much clustering is bad, is there is a sweet spot? Let's explore in a toy grid. Below I've represented four configurations in a 6×6 grid, each with one-third of units marked with a club suit (Figure 7).

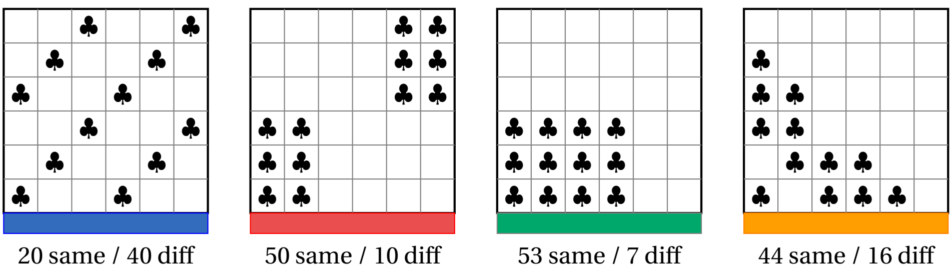


Figure 7: Spatiality matters! For each of these different ways of arranging 12 clubs voters, I've used a "same/different" count to measure clustering. by noting how many pairs of neighbors have matching or different symbols; for instance, 44 pairs of neighbors in the orange grid have the same marking (club-club or blank-blank) while 16 neighbor pairs are different (club-blank). In network science, this kind of same/different statistic is called *assortativity*. This captures something (but not everything) about the geometry of the configuration.

I can try partitioning these into six same-sized "districts" and see how much the layout matters, even while the vote share stays constant. Try it for yourself—some of these symbol layouts give you greater control of the outcome than others. Some spatial arrangements make it possible to lock out the clubs voters from representation entirely; in other arrangements, it's possible to overshoot proportional representation. For instance, I can shut out the blue grid's clubs voters by drawing vertical-stripes districts. The best I can do if I'm trying to *maximize* their representation, on the other hand, is to draw a plan that gives them two districts out of six, and that's not so easy to find. This world of possibility is almost disjoint from the one afforded by the political geography in the orange grid!

How the distribution of clubs votes relates to district outcomes is surprisingly subtle. But in this small example, it's the *most* clustered arrangement (green) that is in line with proportional representation (1/3 of the votes tending to earn 1/3 of the seats), and this is way better than the outcomes I should expect of a typical layout. We can see histograms summarizing *all* the possible ways of districting these grids in Figure 8. The very best layout possible for 12 clubs voters is the one shown in orange—the expectation is actually slightly super-proportional!—but others with a similar clustering score are not as advantageous. The spatial effects are stubbornly multidimensional; political geography is not captured in a single clustering score.[9]

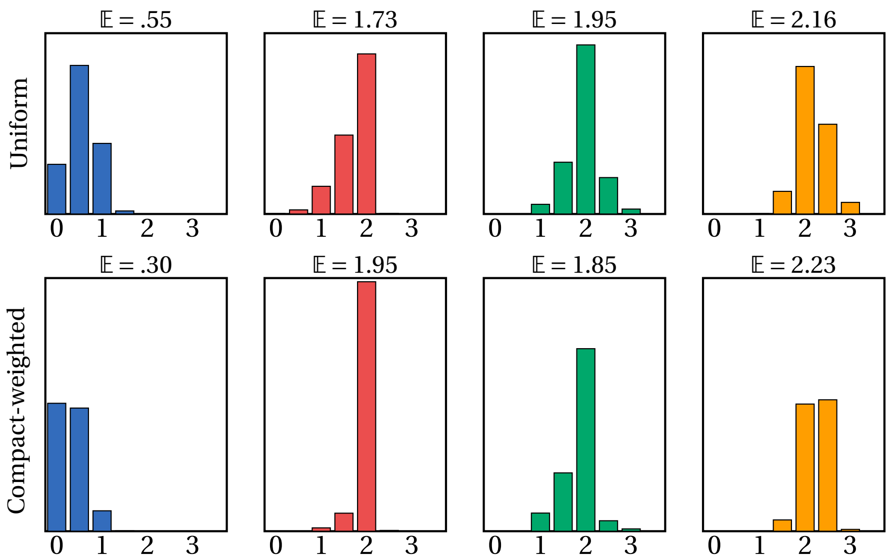


Figure 8: How much can I gerrymander? This plot shows how many seats would be won by the clubs party for every single way of districting the grid—there are 451,206 contiguous plans in all. (I gave clubs credit for .5 of a district if it got three out of six votes.) The top row is uniform: all plans are weighted equally, so for instance the most common outcome on the red grid is that two out of six seats are held by the clubs party. The bottom row shows the exact same set of possibilities, but where plans are weighted according to compactness—plump plans get heavier weight and snakey plans contribute more lightly to their histogram bars. (To be precise, this uses *spanning tree weighting*, which will be explained a bit further below.) So if there's a preference for choosing compact plans, the two-seats outcome becomes overwhelmingly likely on the red grid.

We were able to unearth considerable complexity in the problem by completely enumerating the plans for the $6 \times 6 \to 6$ districting problem. Now consider that I'm not able to construct all the plans even for a 9×9 grid, and I can't even *count* all the possibilities for a 10×10 grid or reasonably *estimate* the possibilities for Pennsylvania's precincts. How can I assess the consequences of the "political geography" to disentangle gerrymandering from the neutral consequences of districting?

[9] Or at least not *this* clustering score (meaning assortativity, or the "same/diff" count shown above), or any that I have seen or tried—see Chapter 15 for more discussion of spatial statistics. It would take a lot of space to provide enough examples to make this point fully, but you can play with spatial effects yourself at mggg.org/metagraph.

2.3 SAMPLING

This sounds like a hopeless state of affairs. We're trying to evaluate one way of cutting up a state, but without any measure of the size of the universe of alternatives, let alone a catalog of its properties to compare against. This sounds like groping around in a dark, infinite wilderness.

The good news is that even universes that can't be definitively mapped can often be effectively explored with random sampling. You don't need to talk to every American to conduct a good poll; you can use statistics from a representative sample to understand the wider universe. To do this well, you'll need to think about weighting and sample size. (We'll return to this below.)

There is a bevy of sampling techniques you might use for redistricting (Chapter 16, Chapter 17). Instead of profiling those, let's stay broad. What it is to be a representative sample in any context?

Building a Sample

Step 1. Come up with relevant categories or types;

Step 2. Construct a raw sample that encounters all relevant types;

Step 3. Re-weight the raw sample to reflect the population you want to represent.

Let's stick with polling to illustrate some of the issues in play. There are a lot of ways to fail as a pollster! Suppose my ultimate goal is to get a sample of intended voters that is representative of the electorate. If my whole poll is conducted by cell phone calling, then I will entirely miss some kinds of people—those who don't have cell phones, or those who don't pick up from an unknown number. If a lot of people hang up on me when they hear my first question, I'll have too few responses from a certain type of voter. In order to counteract the over-representation and under-representation in my raw sample (relative to the electorate), I need to do work to come up with relevant categories, such as "Angry White Guy Who Thinks Coronavirus Is A Hoax" (AWGWTCIAH). I will then need a sense of how much of the electorate is made up of AWGWTCIAH so that I can counteract the skew in my sample relative to the universe I want to represent. That lets me re-weight my raw sample so that AWGWTCIAH voting preferences are in balance.

If you are thinking "Well, I don't know any AWGWTCIAH!"—yes, that is kind of my point. A uniform distribution on your friends and family, or even a uniform distribution on the voting-age public, is not going to give you a sample that represents the electorate. It's easy to miss that a lot of fundamental conceptual work happens in Step 1 and Step 3. It's also easy to forget that if Step 2 fails, so that you *never* encounter certain types, it can't be corrected by re-weighting.

These reweighting ideas are crucially important in redistricting, because there is a type of silly and unreasonable districting plan that wants to dominate your sample if you let it! Namely, there is an over-supply of plans that are so wild and snakey and flagrantly noncompact that they look like fractals and put the original gerrymander to shame (Figure 9).

0.3 BUILDING RANDOM SAMPLES OF PLANS

Markov chain Monte Carlo, or MCMC, is an industry standard across scientific domains for impossible search tasks such as ours. It's a tool capable of decoding ciphertext, probing the properties and phase transitions of liquids, finding provably accurate fast approximations for hard computational problems, and more. (Persi Diaconis's classic 2009 survey [3] estimates that 10–15% of statistical work in science and engineering is driven by MCMC, and the number has probably only gone up since then.)

Essentially, the strategy of MCMC for sampling a collection of objects is just to take a *random walk* in the universe of objects and see what you see. In our case, you can start at any districting plan and make a random transformation to obtain another, then iterate as many times as you like. Then you can compare a proposed plan to the *ensemble* that you encountered on your random walk. It turns out that for many problems where solutions are hard to construct exhaustively, you can still sample quite efficiently if you have a well-designed engine for making those iterative alterations. You're building out what you need from a starting point, using chains of elementary moves.

So you take a million, a billion, or a trillion steps and look at the aggregate statistics. There's mathematics in the background (ergodic theory, to be precise) guaranteeing that if you random-walk in a suitable space for long enough, you'll hit a probabilistic steady-state. This allows you to collect a sample whose properties are representative of the overall universe, typically far before you've encountered every alternative.

I've been involved in developing a family of samplers called "ReCom" (or recombination) that are powered by large moves in the space of plans, and for which we have a good approximate description of how their ensembles ultimately distribute. Heads up: recombination-style samplers do not weight all districting plans equally, *and that's a good thing!* Weighting all plans equally would tend to put far more weight on noncompact districting plans than on plausibly compact ones, just because of how many more ways there are to be snakey than plump.

ReCom works by fusing two whole districts at every step, choosing a district skeleton called a *spanning tree*, then finding a place to snip the tree that leaves two population-balanced pieces behind. Once they converge, ReCom samplers draw from (approximately) the distribution that weights plans according to the number of spanning trees of their districts. (A different elementary move would target a different distribution; you can think of this as the *distributional design* of the random walk.) This distribution is precisely the "compact-weighted" one that produced the club-suit statistics in the bottom row of Figure 8.[a] The spanning tree distribution nicely blends visual compactness, in that it favors fat over spindly districts, and functional compactness, because it favors plans that have more connections *within* relative to connections *between* the districts.

Today the cutting-edge scientific questions concern better theoretical guarantees and convergence diagnostics, as well as efficient implementations. And together with all this, we must keep building persuasive ways of talking about it!

[a]This distribution gives the most compact partitions of the grid over 11 million times the weight of the least compact ones. Chapter 17 and particularly Sidebar 17.8 provide the ingredients to compute this for yourself!

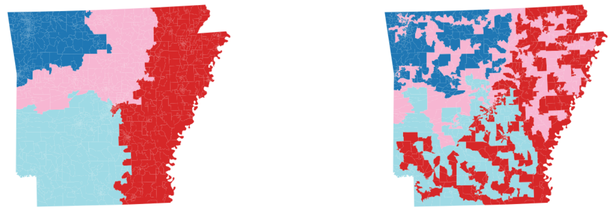


Figure 9: If you don't re-weight, your plans will look crazy! In a uniform sample where all contiguous and population-balanced plans are counted equally, over 99% of plans will look wildly noncompact. If you try to fix this by just setting a worst allowable compactness score, then 99% of plans will be at the worst level you allowed and you will essentially never see compact ones.

So finding good ways to sample while maintaining reasonable compactness is the name of the game—we need to over-sample or up-weight the compact plans in order to collect an ensemble that is plausible for the redistricting application, and we would like to do so by an elegant mechanism that leaves us able to explain why and by how much we weight some more than others. In my lab we've developed a method that weights plans in direct proportion to the compactness of their districts, and only that—no hidden factors. (See Chapter 17.) Then other plan criteria can be layered in. A uniform sample of all plans is both intractable and not that valuable; a *representative* sample of plausible, valid plans is the goal.

3 THEORY MEETS PRACTICE

What happens when we scale up from a small grid to, say, Pennsylvania? That is, how would plausible redistricting plans tend to look if drawn without a partisan agenda? Let's be careful: that's a really different question from locating partisan fairness, let alone living up to our highest ideals of representative democracy. And it allows us to stay in *descriptive* mode (what do plans made with no election data look like in partisan terms?) rather than making *normative* declarations (this is how party spoils "should" be distributed).

3.1 THE DATA

First, real-world modeling demands the collection and preparation of data. And, as always, there's a whole story to be told about that. Throughout this book and all of the debates about how best to study gerrymandering, there's a fundamental need for a certain kind of data that is very hard to get.

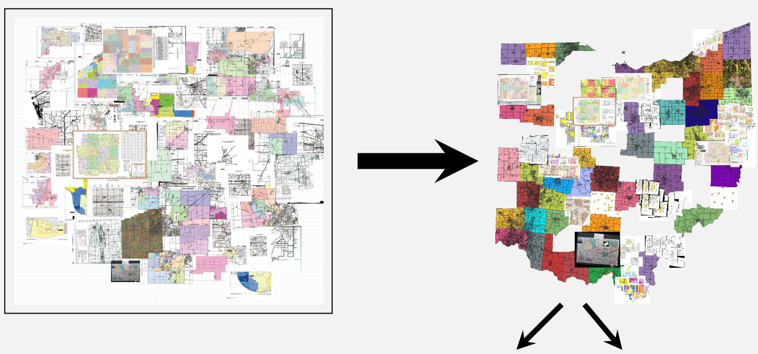
Americans vote in *precincts*, which are geographical units for election administration that usually have a population of a few hundred to a few thousand. (Typically, each precinct has a single polling place, but it's sometimes several-to-one in either direction.)

For instance, the math department at Tufts is in a small building that has two entrances, each in different precincts. The main entrance is in one precinct (population 3902 in the 2010 Census), and the back entrance is in another (population 3567). Election results are typically reported in cast vote totals per precinct. So the MA Secretary of State website tells us that the front-door precinct went 1270–120 for Clinton over Trump in 2016, while the back-door precinct went 1013–167. What's more, those precincts are in different cities and different Congressional districts! The front door is represented by Katherine Clark and the back door is represented by Ayanna Pressley.

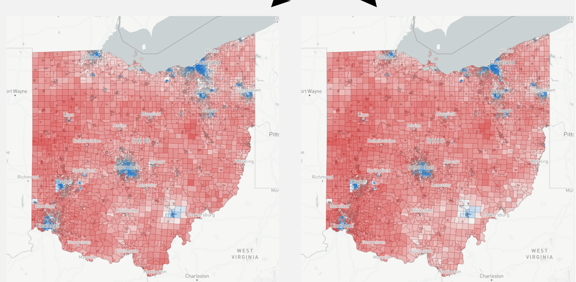
0.4 THE PRECINCT PROBLEM

At the Voting Rights Data Institute in 2018, a team led by Ruth Buck, Katie Jolly, and Katya Kelly put dozens of students to work on figuring out the precinct boundaries from the 2016 elections in Ohio. We called all 88 counties to ask the simple question: can you send us a map of your precincts? 46 counties sent shapefiles, which are the industry standard digital format for spatial data. 27 counties had PDF maps. 8 counties sent paper maps (including highway maps with marker and tape!). 7 had nothing. We spent hundreds of person-hours digitizing and georeferencing the maps to build a statewide shapefile.[a] And that was for one year's elections!

So whenever you see highly granular color-coded maps of election results, think twice about the accuracy of the dataset. (I'm looking at you, New York Times![b])



These look nearly the same to the eye, but we'll find that they give very different districted outcomes! (Figure 10)



President 2016 vote 45.74% D / 54.26% R

Senate 2016 vote 39.04% D / 60.96% R

[a] The artistic rendering here is by Emilia Alvarez. See github.com/mggg-states for details.

[b] "An Extremely Detailed Map of the 2016 Election," 25 July, 2018.

Since the precincts are where the votes are reported, you really need to know where they are located to analyze the impact of districting lines. But believe it or not, in most states in the country, *nobody knows where all the precinct boundaries are at any given moment.* That's because local election officials—usually county officials, but in some states like MA and WI it's actually town officials—have the authority to administer elections and to change the precincts, and in many states they have no reporting requirements, so even the secretary of state is not kept abreast of changes.[10] For the rest of the chapter, we'll restrict ourselves to states where it is reliably reported, or where someone has painstakingly assembled it.

3.2 POLITICAL GEOGRAPHY

Data in hand, we'll churn out a party-blind sample of plans and see how they cut up the votes into representation. This gives us a capsule summary of the consequences of political geography. For the following three sets of three states, we've built 50,000 random plans that are compact, contiguous, and population-balanced for each state. Each of these states had a U.S. Senate election in either 2016 or 2012; I have laid the random plans over the voting pattern for the Senate election and for the Presidential election the same year. For each plan in the ensemble, I then report the same summary statistic: how many districts have more Democratic than Republican votes? The outputs show us the "mere" mathematical consequences of single-member districts interacting with recent American political geography.

What can we see at a glance? I see two forces here, sometimes aligned and sometimes in opposition.

- A tendency for districted systems to underrepresent minorities, in this case minority party preferences;
- A Republican advantage.

In the states that lean Republican overall (Figure 10), these point in the same direction and combine to create a world of districting plans that favors Republicans by several seats, so that a proportional outcome (marked by the red line) is rarely observed under blind districting. This is most dramatic in Ohio's Senate pattern, where even a 25% (4 out of 16) showing for Democrats is rare, despite their 39% vote share. It is fundamentally important to note that this does not mean that one couldn't draw a plan with six Dem seats—on the contrary, it is sometimes quite easy for a person to draw by hand a plan with properties never observed in an ensemble. But in the world neutrally constructed by the rules, if Ohioans cast votes in the pattern that they did for Rob Portman against Ted Strickland, but it would probably take more than the lifetime of the candidates for Team Strickland to draw a map from a neutral process that would give him a proportional showing.

[10] I'm really not exaggerating. See for instance this court filing by the Pennsylvania legislative leaders (perma.cc/ECU9-PATG), or this statement by the Virginia Division of Legislative Services (perma.cc/KE85-3A95).

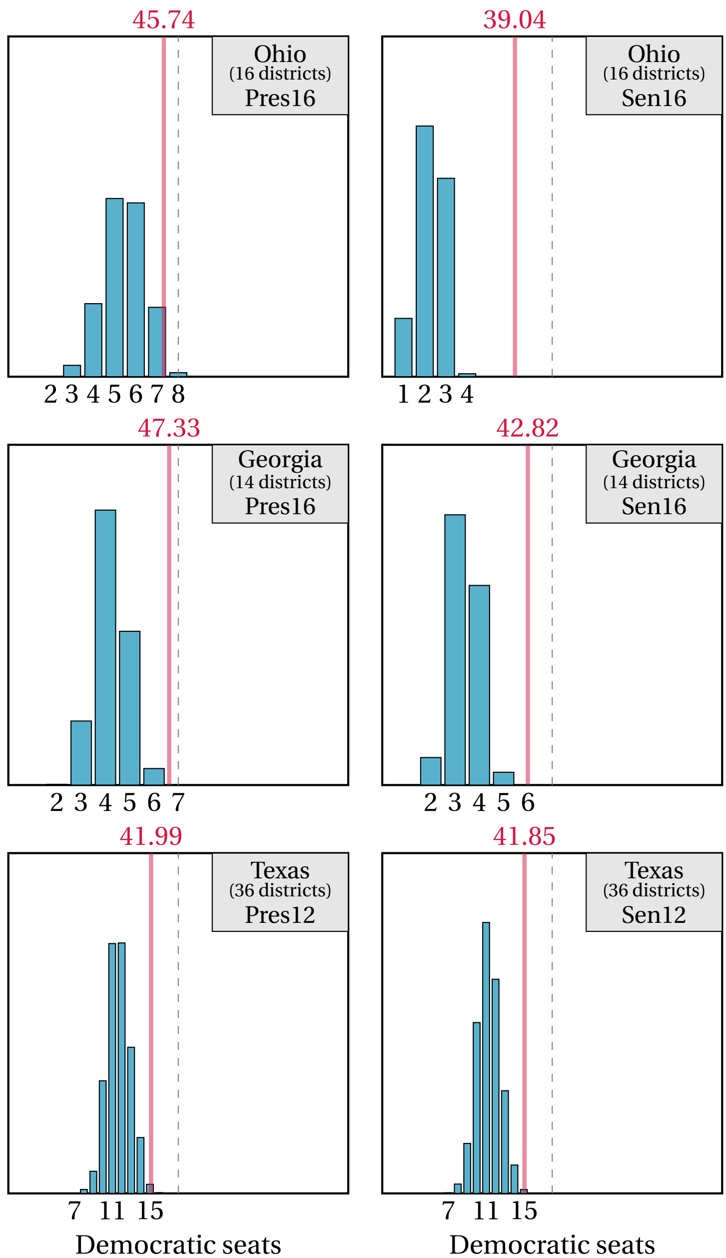


Figure 10: **Consequences of political geography**: how many Democratic-majority districts would result from "blind" redistricting (using no partisan data) in Republican-leaning states? The box at top right tells you what state, how many districts, and which statewide vote pattern is used in the background. The dotted line is placed at 50–50, while the red line marks the statewide Democratic vote share in that election. The histogram shows a neutral ensemble of 50,000 compact, contiguous, population-balanced plans made without partisan data. For example, we observe that a proportional outcome for Democrats in Texas Congress would be 15 seats for either 2016 vote pattern (see red line), but random plans (in blue) very rarely achieve that.

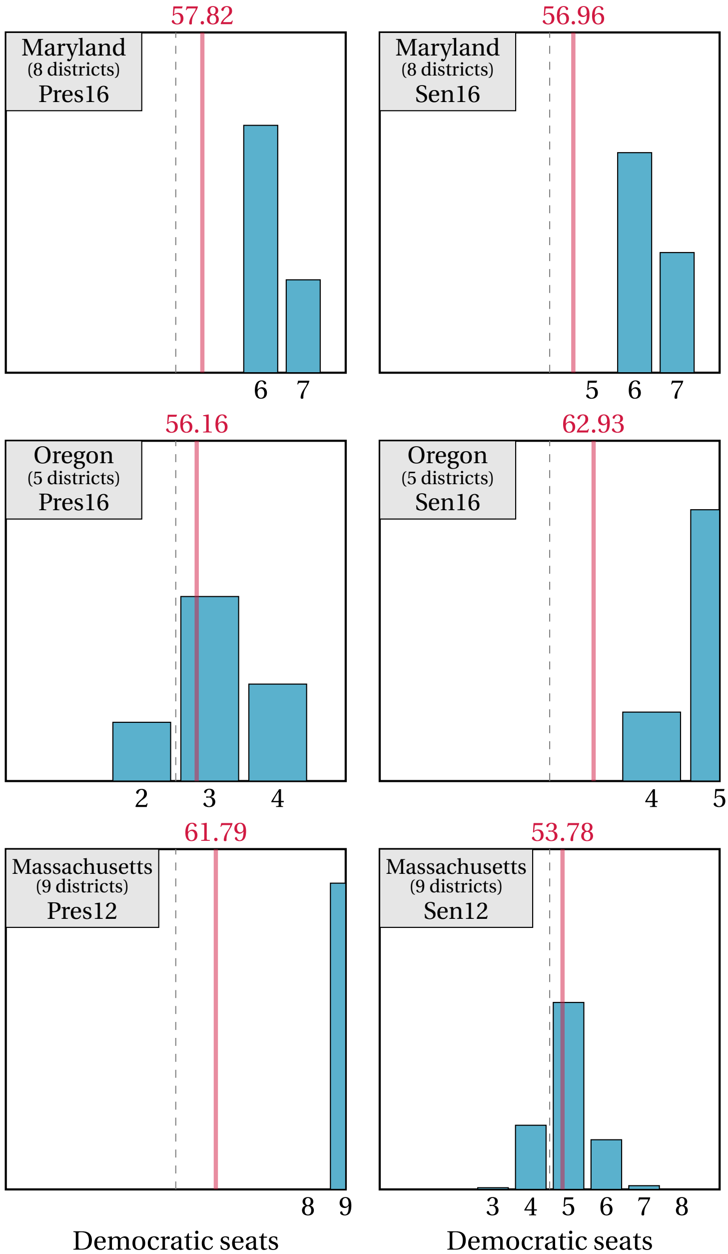


Figure 11: **Consequences of political geography**: how many Democratic-majority districts would result from "blind" redistricting (using no partisan data) in Democratic-leaning states? The box at top left tells you what state, how many districts, and which statewide vote pattern is used in the background. The dotted line is placed at 50–50, while the red line marks the statewide Democratic vote share in that election. The histogram shows a neutral ensemble of 50,000 compact, contiguous, population-balanced plans made without partisan data. For example, we observe that more than half the plans in the ensemble give a proportional outcome in Massachusetts with respect to Senate voting, but no plans get close to proportionality for the Presidential vote pattern. This squares with the earlier observation that elections with two-thirds/one-third vote preferences in Massachusetts produce a district lockout for the minority party.

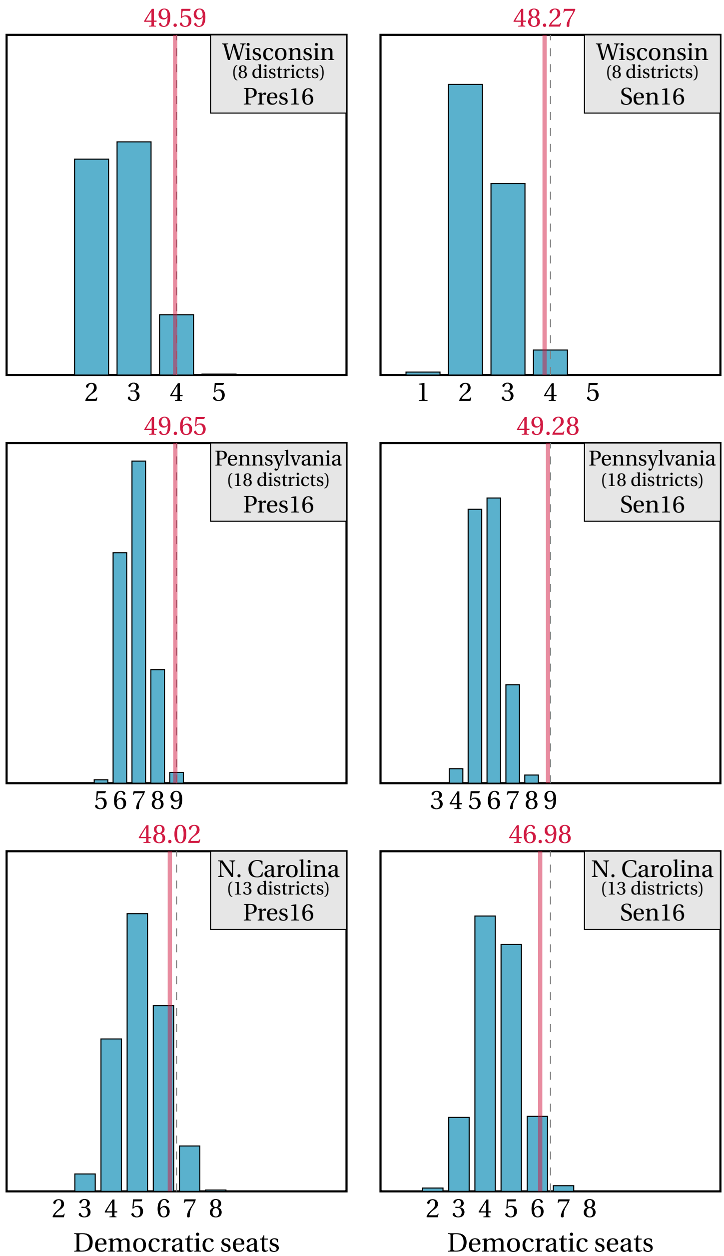


Figure 12: **Consequences of political geography**: how many Democratic-majority districts would result from “blind” redistricting (using no partisan data) in very close states that were litigated as Republican gerrymanders? The box at top right tells you what state, how many districts, and which statewide vote pattern is used in the background. The dotted line is placed at 50–50, while the red line marks the statewide Democratic vote share in that election. The histogram shows a neutral ensemble of 50,000 compact, contiguous, population-balanced plans made without partisan data. All of these examples show a tilt by several seats in the Republican direction, with respect to proportionality.

The outcomes in Democratic-leaning states (Figure 11) are consistent with a tension between these forces; look at Oregon/Pres16 and Massachusetts/Sen16, for instance, where a small but substantial lead for the Democratic candidate has not reaped the same benefits for Democrats as it did for Republicans in the earlier set of examples.

In the last set (Figure 12), we see that razor-thin statewide margins do not induce histograms centered around a 50-50 delegation. Since the races are so close to even, the explanation must be spatial, having to do with where the votes fall and not just with their overall balance. This, then, is the political geography in action.

Why the anti-minoritarian outcomes? Once this is observed, it's not that hard to explain the math: having 1/3 of a population, say, does not translate to having greater than half of a random sample 1/3 of the time. This produces consistent structural disadvantage for minority groups in districted systems. Smaller groups can only be rescued by spatial concentration, which lets a global minority appear as a local majority. These effects were previewed with the club-suit voters of Section 2 above—only the most clustered configurations gave them ready access to good representational outcomes.

The precise causes of the partisan lean are harder to diagnose. For one thing, our discussion of the benefits of clustering is directly at odds with the conventional wisdom about Democratic disadvantage in contemporary American districting, where many authors have blamed *overconcentration* for Democratic shortfalls. (Namely, through the heavy Democratic lean of urban areas.) Personally, I have not yet heard a specific mechanism proposed for city effects that is amenable to modeling. So I think this remains one of the tantalizing open questions in the field: how does the human and political geography of many U.S. states in the 21st century interact with the mathematics of districts? And how do the features combine to produce such a gravitationally tilted playing field for the two major parties?

3.3 REASONING FROM ENSEMBLES

As promised, we've built a fairly powerful descriptive account of redistricting. Using hard-won datasets and carefully designed sampling techniques, we've set ourselves up to understand the neutral tendencies when districts divide up territory, letting the chips fall where they may against American voting patterns.

From here, there are several normative moves we can make—that is, we have some choices about what we'll regard as fair. One simple idea is to flag outliers as *unfair*. For instance, the wild-looking Pennsylvania plan from 2011 gives four Democratic seats when you lay it over the Senate 2016 voting pattern, putting it in a tail of the curve, and indeed in the tail that's most favorable to the party in charge. And the plumper and lovelier Twitter map? Also four.

It should not be surprising that all of three of the purple states from Figure 12 saw legal challenges on the basis of partisan imbalance (though Wisconsin's focused on legislative rather than congressional districting); the world is tilted in a Republican direction, and Republican legislatures still chose from the "far side of the bell curve" in each case.[11] They leaned in to their structural advantage.

But let's remember that taking up a negative norm (like "don't be an outlier") does not commit you to a positive norm (like "you must look perfectly typical"). The fact that 7 seats and 6 seats are at the peak of the bell curves for those election patterns in Pennsylvania can perhaps help us pull apart the effects of careful partisan design from the consequences of the system itself. As such, it might help us flag a gerrymander.[12] But are those neutral plans our best choices for a healthy democracy?[13]

4 ADDING THINGS UP

Proportionality along party lines is a ready benchmark, but it's not the only one that we might select. There are many alternatives, such as wanting every sizeable bloc and region to be reflected in the legislature; ensuring rotation of control by engineering for competition and responsiveness; promoting continuity in representation when it has proven effective; avoiding gridlock by buffering a small vote advantage to a more secure governing advantage; excluding violent extremists; and on and on. We can certainly achieve some suite of good-government goals in a districted system, especially when all the extrinsic factors are favorably aligned—but our chances of that are better when the lines are drawn by those who are well informed about the alternatives, and whose incentives align with legitimate societal goals.

We should never confuse neutrality with fairness, in any context, especially when the process itself is up for grabs. That is, we have to keep thinking and debating about our representational ideals and requirements—and seeing how best to draw districts that measure up. At some point that might even be too big of a strain, making alternative voting systems more attractive. We never signed up for blind districting, and even districting itself wasn't graven on tablets as a fundamental form of American government. As the properties of districting become clearer, we should keep asking whether and how we can get good outcomes, whatever we take good outcomes to be.

The sum total of some years of obsessive study of this stuff has left me with some big-picture views, many of which will be explored (or challenged!) by the authors in this volume.

[11] Relative to (Pres16,Sen16) voting, the legislatures' enacted plans give these Democratic party outcomes: Wisconsin (2,2), Pennsylvania (6,4), and North Carolina (3,3). The state courts ultimately issued replacement plans: Pennsylvania (8,5) and North Carolina (5,5). The Wisconsin case was federal and hit a dead end.

[12] As my friend and collaborator Jordan Ellenberg memorably put it not so long ago, "The opposite of gerrymandering isn't proportional representation; the opposite of gerrymandering is not gerrymandering." *The Supreme Court's Math Problem*, March 29, 2019. perma.cc/R6VJ-CDPM

[13] This question takes on a whole different fairness dimension applied to racial and ethnic minorities instead of mutable party preferences; see [2] for an extended discussion of the merits of race-blind redistricting.

4.1 THINGS I DON'T BELIEVE IN

1. **Presenting any single number as a metric of fairness.** Given all of the complexity of balancing multiple objectives, you might say that the premise that fairness could be captured in a single statistic is implausible on its face. After years of studying the various attempts to score fairness, I can tell you that it's unsupportable on closer review as well. This does not doom our ability to handle it well. We have a system for deciding when there's been a murder, and we're comfortable with the fact that this requires complex, interdisciplinary evidence that could come from chemistry, ballistics, psychology, etc. We don't expect it all to be captured in a "murder score."

2. **Especially any single number with a prescribed ideal.** People often feel that a non-gerrymandered map would have such-and-such a property in a state of nature. For instance, we've spilt a lot of ink on (a) the nearly universal intuition that neutral districting attains rough proportionality, and (b) the finding that it does not. It might be just as tempting to believe, for instance, that a non-gerrymandered map would have a roughly equal number of "wasted votes" for each party.[14] Likewise, if a certain election had a statewide average of 60% for one party, we might expect that a truly neutral outcome would have the same number of districts over 60% as under 60%.[15] But districted plurality outcomes are a highly geometrized and nonlinear affair, and there's no reason at all for these ideals to obtain in the absence of gerrymandering. Instead, if you want to know if a score is good, there's just no reasonable way around comparing to the others that are actually achievable in your particular setting.

3. **Redistricting as optimization.** When many technical people take up the redistricting problem, they import a paradigm that I think is unhelpful, which is to look for an *objective function* (some sort of grand unified score of goodness) and seek a plan that *optimizes* it. But redistricting never has been—and shouldn't be—a literal matter of finding the best map, even if you *did* have a reliably informative score in hand. The whole project of drawing good territorial districts is about capturing community while working with an eye to representational balance, and this means it's properly a human and holistic affair. We wouldn't and shouldn't be satisfied that one plan is chosen over another qualitatively very different one because its score is better in the fifth decimal place. Computational redistricting can help people to understand tradeoffs and generate novel alternatives, but it works best as assistance rather than outsourcing.

4. **Redistricting as a game.** While computer scientists reach for optimization, economists often reach for game theory as a paradigm to make sense of fairness. Here, the state might be regarded as a resource that should be divided "fairly" among game-playing agents.[16] But this typically sets up redistricting as an adversarial game between the political parties, and all the other qualities that make a plan matter fall away to ancillary status.

[14] This is the efficiency gap test.
[15] This is roughly what is called the mean-median test.
[16] There is really some beautiful math in this direction. See Pegden–Procaccia–Yu, Landau, etc. [4, 5]

5. **Confusing rules of thumb for laws of nature.** When things are hard to model, it's totally reasonable, even essential, to use rules of thumb. But often a shortcut gets elevated to a method and the simplifications that made it useful turn into conceptual baggage. From "uniform partisan swing" to "the seats-votes curve" to a "vote index" that captures the sum total of a state's likely voting behavior, this field is full of those. Interdisciplinarity can sometimes provide needed conceptual independence from standard constructs, making space for novel ideas.

6. **Gerrymandering tests only a computer can pass.** (Or even worse, gerrymandering tests that only your own algorithm can pass!) This is a clear risk when computational indicators are elevated as badges of fairness. For instance, imagine that we are presented with a human-generated string of 0s and 1s and we're trying to judge if they were generated without a bias towards one kind of digit. That is really different from asking if they are statistically indistinguishable from binary seqeunces made from random number generators. It's well known that people are bad at imitating coinflips: a human trying to be fair is much more likely to write 01001010111011001010 than 01111100011100100001 even though they are equally likely for a perfect uniform coinflip process. Anomaly detection is important, but it's a whole lot less useful if it devolves into mere *human detection*. To avoid this, it's essential that gerrymandering tests be "ground-truthed" on plans that were made without gerrymandering intent.

7. **"Democrats pack themselves."** As I've argued above, the conventional wisdom around urban disadvantage is underdeveloped, and identifying mechanisms for this should be a big research question in the coming years. Meanwhile, the good news is that we can model the effect, and identify its magnitude.

8. **The tyranny of the median.** So you've got an ensemble of alternatives! What do you do with it? Demanding that a plan should fall right in the middle of the distribution to be deemed fair is a little bit like demanding that a coin should have exactly 50 heads in 100 flips. If you get 54 heads, we shouldn't conclude a slight pro-heads bias; that is well in the normal range.

4.2 THINGS I DO BELIEVE IN

1. **Arguing from alternatives.**
2. **Ranges, not ideals.**
3. **Using quantitative information to tell a qualitative story.**
4. **Mathematical modeling of rules and their consequences.**
5. **Community.**
6. **Humans.**

5 CONCLUSION: WHAT'S NEXT?

In working on redistricting, I have tried to look for the places where mathematical thinking and modeling can make an intervention that helps people to understand and clarify their representational goals. This has also led me to identify some contexts in which districts are just not the right tool for the job, and to study other systems of election (particularly ranked choice voting) that may better promote shared goals and ideals of fairness.

In the meantime we can work to build better districts. Since the last big decennial redistricting, quite a few states have shifted the way they draw the lines: voters in Colorado, Michigan, Missouri, Ohio, Utah, and Virginia all approved redistricting reform at the ballot box. As we've watched new independent or bipartisan commissions get spun up all over the country, they differ in the kinds of help they are seeking. Some of them are sticking with old-school consultants, or even dueling consultants (Democratic and Republican). Others are calling in the kinds of analysis you've seen introduced here, hoping to see where their plans fall in the world of possibility. At the same time, Congress is debating voting bills that contain elements of redistricting reform. Then there's the looming inevitability of litigation. In all of these settings, fancy algorithms and shiny metrics will be leveraged for litmus tests and beauty contests just as much as for measured analysis.

The spirit of this chapter, and I hope of this book as well, is to use the best available tools and perspectives from many fields to help us understand and improve this enigmatic, deeply American, and now widely mistrusted electoral device: plurality districts.

How should we govern ourselves? Who should represent us? How should we elect? To answer these questions requires reflection, not (just) calculation. With law and computing and geography and political thought colliding productively, and with reform energy all over the country, the 2021 redistricting cycle stands to be an innovative and exciting one.

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