Hospital–University Transport Problem

MGGG Redistricting Lab

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This short writeup specifies the mathematical formulation we are using in our Campus Coronavirus Response project.

Parameters

- n_H - Number of hospitals.
- n_U - Number of colleges and universities.
- D - Matrix of distances between hospitals and educational institutions (dimension $n_H \times n_U$).
- α - The relative cost per distance unit of transporting medical personnel (versus patients).
- β - Maximum dorm bed utilization ($0 \le \beta \le 1$), based on suitability of university facilities.
- $\vec{d}$ - Total dormitory bed capacity at each educational institution (dimension n_U).
- s_H - Splitting parameter: maximum number of hospitals that can send people to a single campus.
- s_U - Splitting parameter: maximum number of universities that can receive people from a single hospital.
- $\vec{p}$ - Demand vector for beds for non-COVID sick patients, indexed by hospital (dimension n_H).
- $\vec{c}$ - Demand vector for beds for patients recovering from COVID, indexed by hospital (dimension n_H).
- $\vec{m}$ - Demand vector for beds for medical personnel and first responders, indexed by hospital (dimension n_H).

Decision variables

- P - Flow of non-COVID patients from hospitals to universities (dimension $n_H \times n_U$).
- C - Flow of recovering COVID patients from hospitals to universities (dimension $n_H \times n_U$).
- M - Flow of medical personnel and first responders between hospitals and universities (dimension $n_H \times n_U$).

Problem

$$
\begin{aligned}
\min_{P,C,M} \quad & \sum_{i\in[n_H]} \sum_{j\in[n_U]} D_{ij}(P_{ij} + C_{ij} + \alpha M_{ij}) \\
\text{s.t.} \quad & P \geq 0, C \geq 0, M \geq 0 \\
& \sum_{i\in[n_H]} P_{ij} + C_{ij} + M_{ij} \leq \beta d_j \quad \forall j \in [n_U] \\
& \sum_{j\in[n_U]} P_{ij} = p_i \quad \forall i \in [n_H] \\
& \sum_{j\in[n_U]} C_{ij} = c_i \quad \forall i \in [n_H] \\
& \sum_{j\in[n_U]} M_{ij} = m_i \quad \forall i \in [n_H]
\end{aligned}
$$

Additional constraints:

$$
P_{ij} + C_{ij} + M_{ij} \leq (p_i + c_i + m_i)\, x_{ij} \quad \forall i \in [n_H],\ j \in [n_U] \tag{1}
$$

$$
\sum_{j\in[n_U]} x_{ij} \leq s_H \quad \forall i \in [n_H] \tag{2}
$$

$$
\sum_{i\in[n_H]} x_{ij} \leq s_U \quad \forall j \in [n_U] \tag{3}
$$

$$
x_{ij} \in \{0,1\} \quad \forall i \in [n_H],\ j \in [n_U]. \tag{4}
$$

The initial constraints ensure that the total flow to each university does not exceed the available beds, while the hospital need for personnel and patient need is met.

Constraints (1) introduce dummy variables x_{ij} (required to be binary in (4)) that are turned on when there is a postive flow from hospital i to university j. This sets up constraints (2) to ensure that each hospital sends to at most s_H universities, and (3) to ensure that each university receives from at most s_U hospitals.

Solving

This formulation specifies a linear program that is handled by the commercial solver gurobi extremely quickly. (With our current Massachusetts dataset, including $n_U = 66$ universities and $n_H = 118$ hospitals, we solve for P and M in less than a second. If we additionally enforce a minimum of 30 beds used at each university that receives people from hospitals, the runtime increases to ~ 3.5 seconds.) The outputs are not integers, but we round them before repoerting them in the model, rather than running it as a full integer program.

See github repo (`https://github.com/mggg/covid-analysis`) for further details and parameter values for initial runs.